ECSE 4962 Control Systems Design

A Brief Tutorial on Control Design

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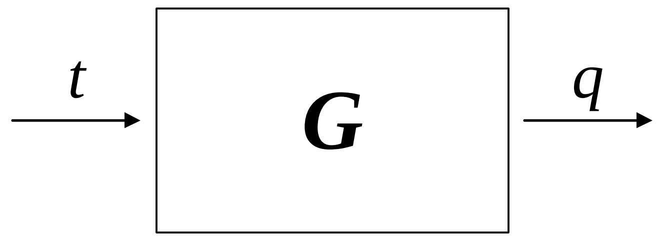
Don't Wait Until The Last Minute!

- **You got to have a model to work with by now:**
  - **At least a model based on estimated mass/inertia and your motors and gears (very few groups have this in the proposal)**
  - **You should very soon have an identified model.**
  - **If you don't, you must seek help!**
- **Once you have a model, start control design – in simulation. Don't just tweak the controller with the experiment without some simulation based analysis first!**

Today

- **Review basic control design**
- **PID tuning**

LTI System Characterization

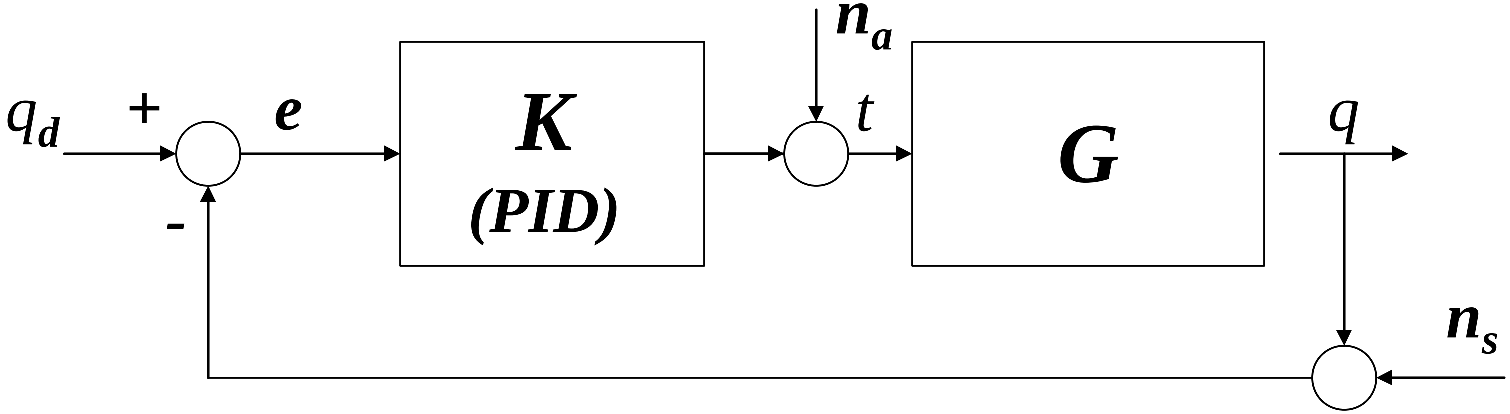


We'll approximate each axis as an independent single-input/single-output (SISO) system: $q = G\ t$

Characterization:

- **poles and zeros** `zpk(G);`
- **frequency response** `bode(G)`
- **step response** `step(G)`
- **steady state value** `evalfr(G,0)`

Closed Loop System



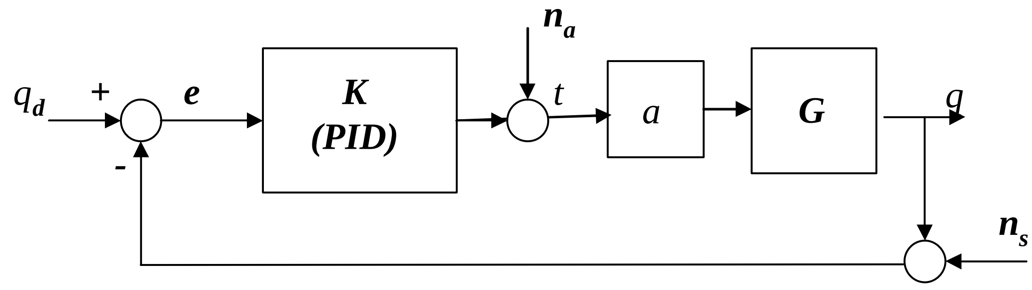
Relevant transfer functions:

$$q = \frac{KG}{1+KG}(q_d - n_s) + \frac{G}{1+KG} n_a$$

$$e = \frac{1}{1+KG} q_d + \frac{KG}{1+KG} n_s - \frac{G}{1+KG} n_a$$

Design Objective

- Stability: closed loop poles must all be in left half plane.
- Performance:
  - Step response has small overshoot and small settling time.
  - Small steady state error
- Disturbance Rejection: Effect of sensor and actuator noises small on q
- Robustness: How large an uncertainty a can be tolerated (in terms of stability)?



Design Objective

- **Stability: closed loop poles must all be in left half plane.**

$$\text{closed loop poles} = \text{roots of } \left(1 + KG\right)$$

Design Objective

- **Performance:**
  - **Step response has small overshoot and small settling time.**

$$\text{closed loop transfer function: } G_{CL} = \frac{GK}{1+GK}$$

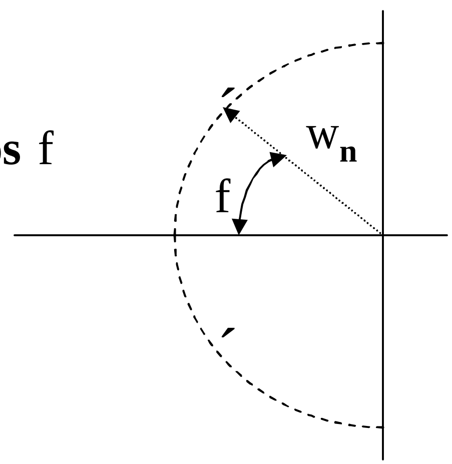
w_n sufficiently large, z sufficiently close to 1

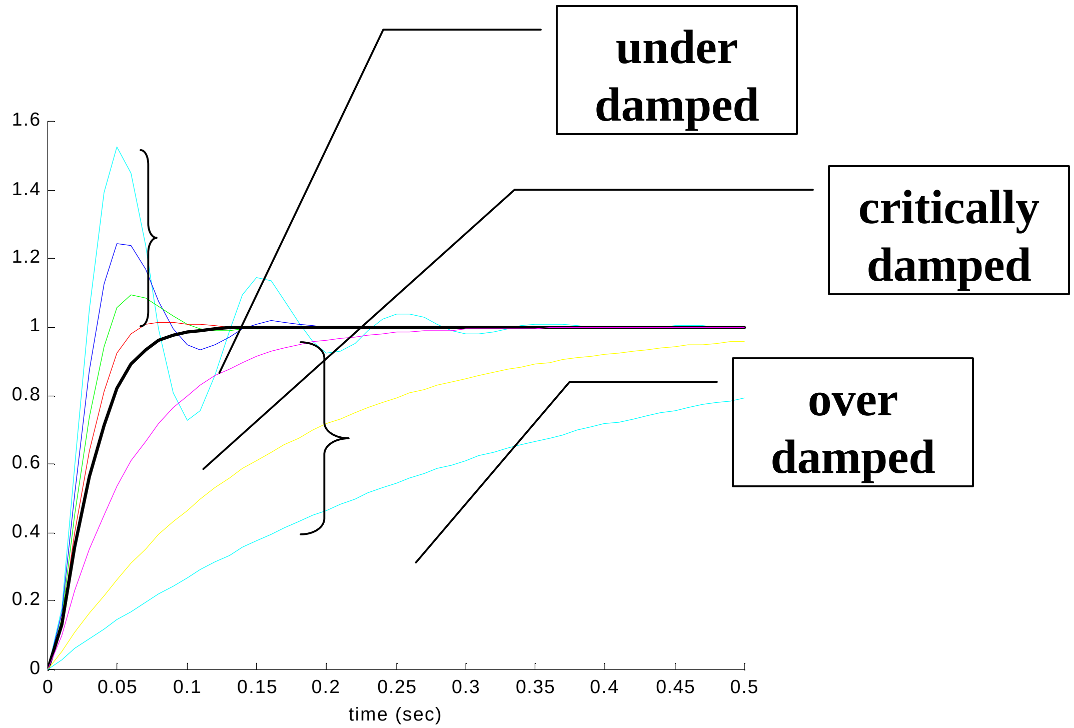
Step Response

- Intuition based on second order system with no zero

$$H(s) = \frac{w_n^2}{s^2 + 2zw_n s + w_n^2}$$

z=cos f

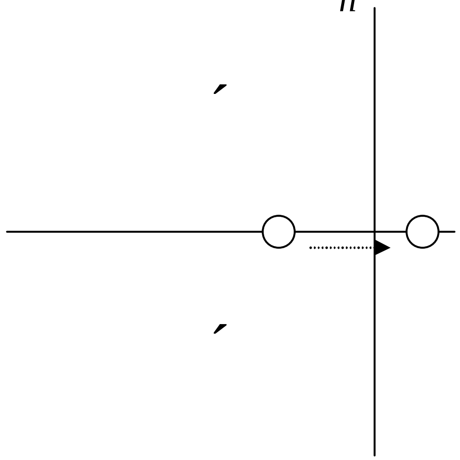


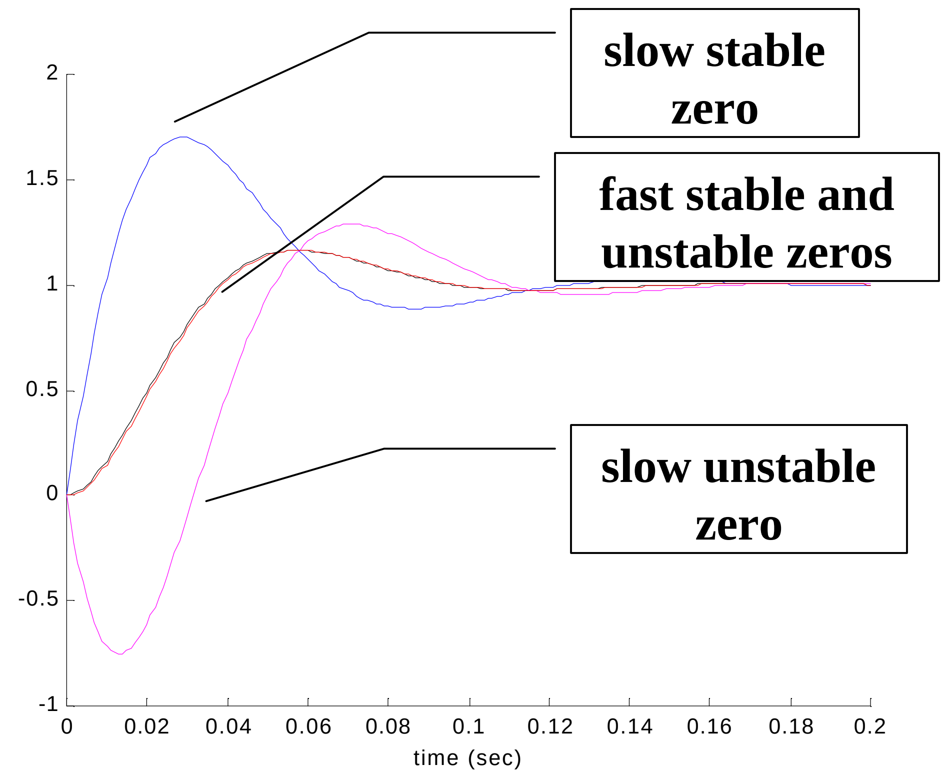


Effect of Zero

- Zero within system bandwidth strongly affects response
- Stable zero increases overshoot, unstable zero gives rise to undershoot

$$H(s) = \frac{w_n^2 (s/a + 1)}{s^2 + 2zw_n s + w_n^2}$$





Design Objective

You want this close to 1

- **Performance:**
  - **Small steady state error**

$$q_{ss} = G_{CL}(0)(q_d - n_{s,DC}) + \frac{G(0)}{1+G(0)K(0)} n_{a,DC}$$

$$G_{CL} = \frac{GK}{1+GK}$$

You want this close to 0

To reduce steady state error:

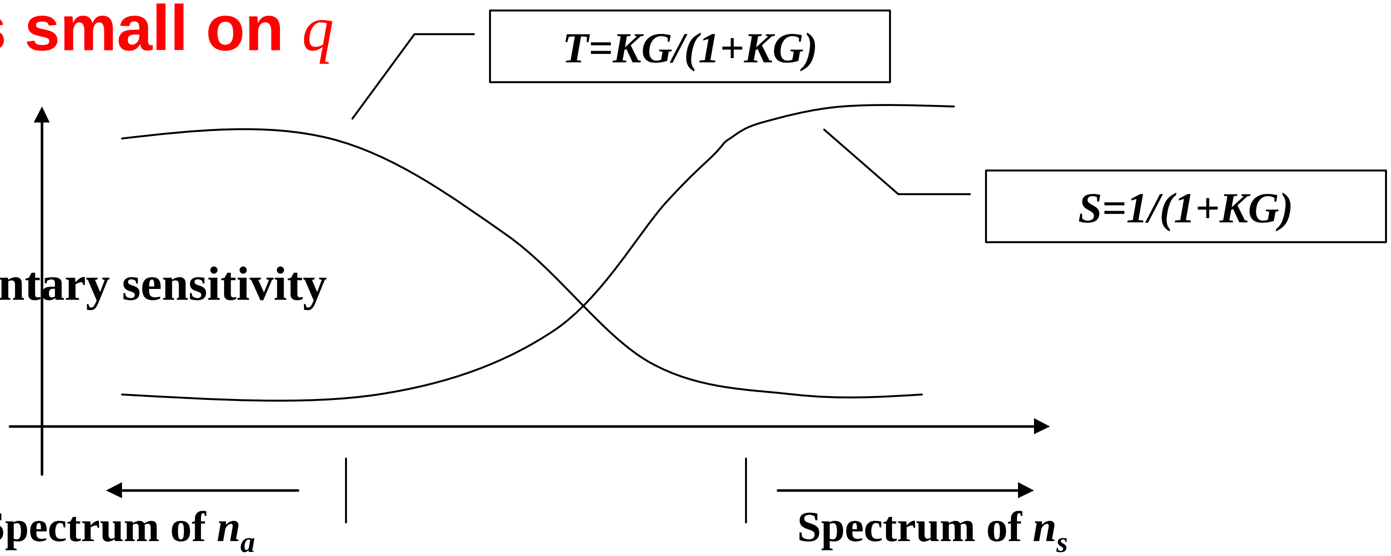
- Cancel n_a if possible
- High DC loop gain $G(0)K(0)$ ➔Integral control

Design Objective

$$e = \frac{1}{1+KG} q_d + \frac{KG}{1+KG} n_s - \frac{G}{1+KG} n_a$$

Design *KG/(1+KG)* small over the spectrum of n_s and *G/(1+KG)* small over the spectrum of n_a

- **Disturbance Rejection: Effect of sensor and actuator noises small on q**



S **= sensitivity**
T **= complementary sensitivity**
S **+ *T* = *1***

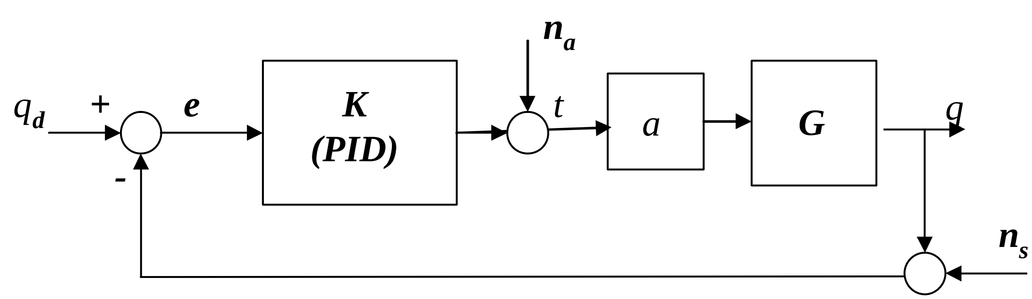
Design Objective

Gain margin: if a **is a real number, how far can** a **be different from 1 before the closed loop system becomes unstable?**

Phase margin: if a***=exp(j****q****)*** **(a pure phase shift), how large can** q **be before the closed loop system becomes unstable?**

Usually expressed in terms of dB:
$a \in$ **[.5,2] means GM=6dB**

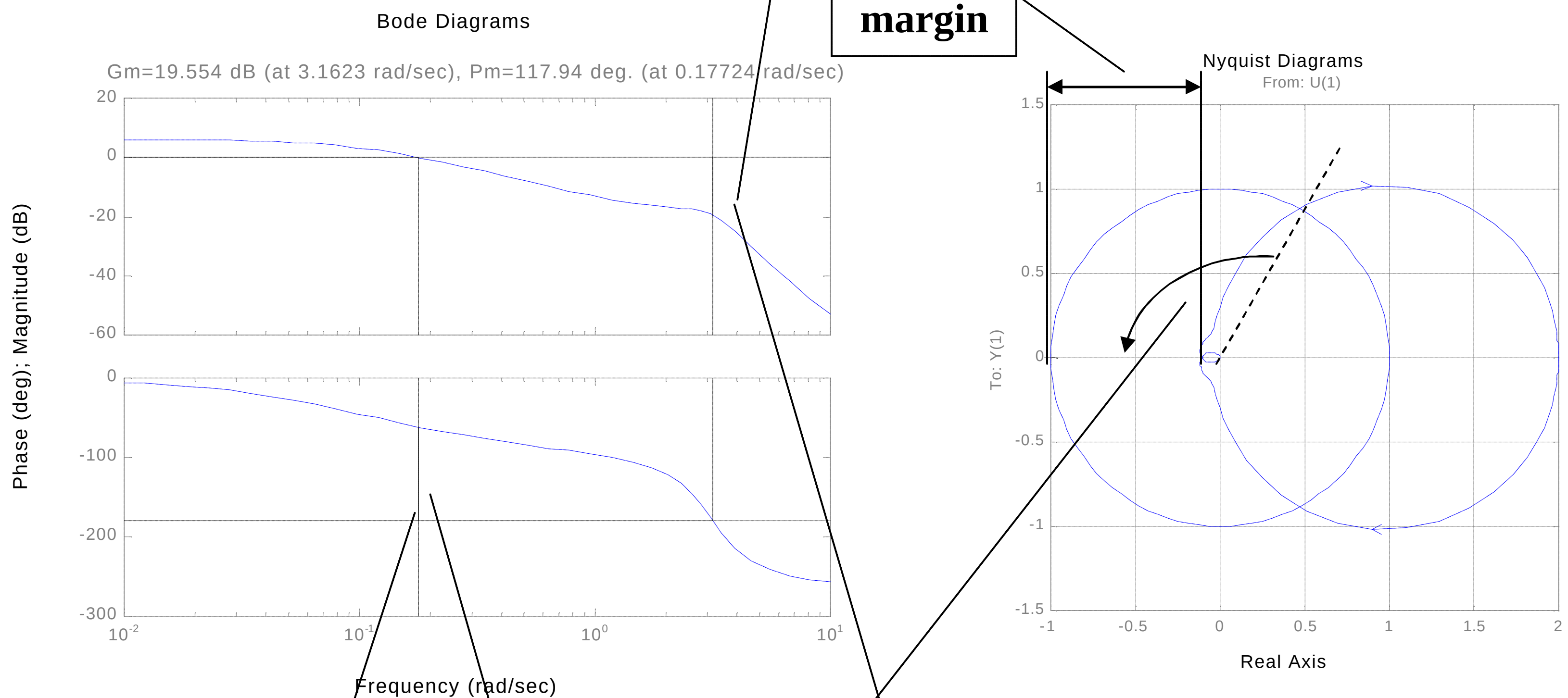
- **Robustness: How large an uncertainty** a **can be tolerated (in terms of stability)?**



Robustness (Gain/Phase Margins)

a is nominally 1

gain margin



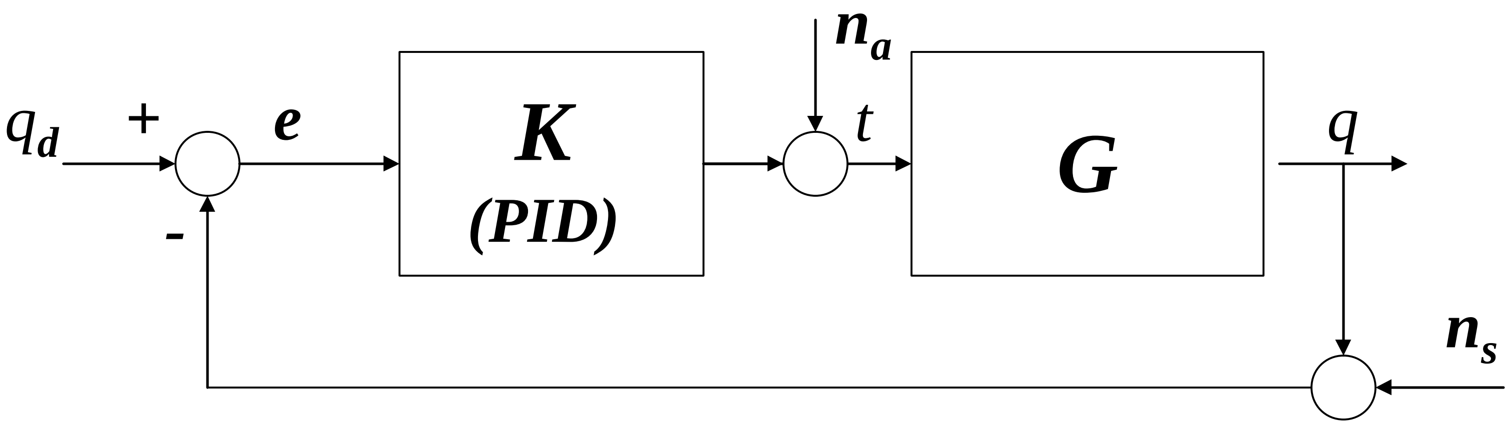
Add phase lead for phase stabilization

phase margin

reduce gain for gain stabilization

use `margin` command in MATLAB

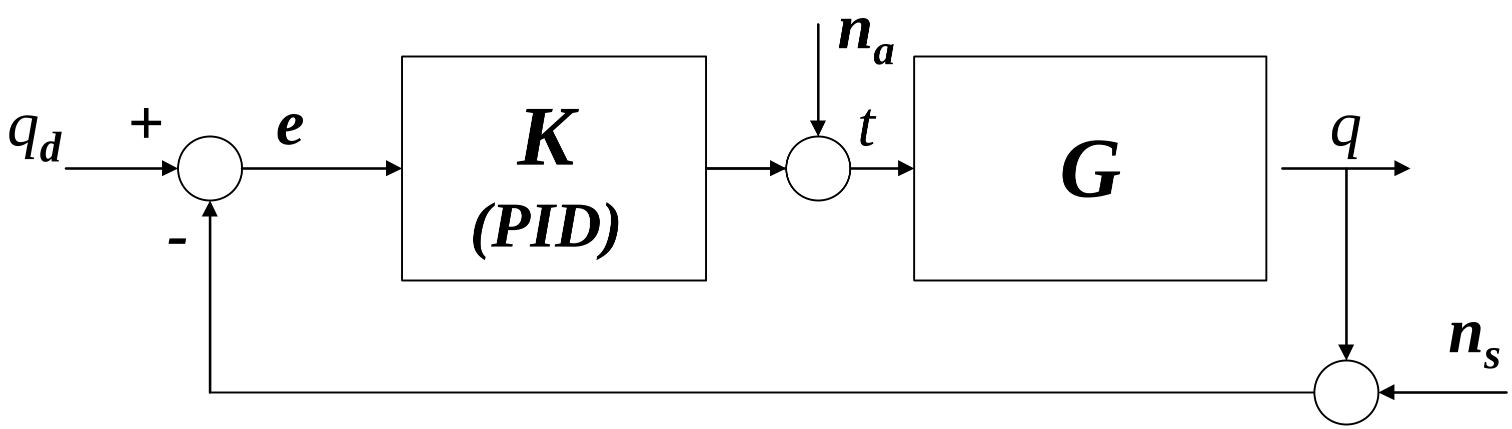
PID Control



$$K(s) = k_P + k_I / s + k_D s$$

When does it work?

PID Control



$$K(s) = k_P + k_I / s + k_D s$$

Works well when *G* is a 2nd order system.

PID Control

Consider $G(s)=1/s^2$: closed loop characteristic polynomial is

$$s(s^2 + K_D s + K_P) + K_I$$

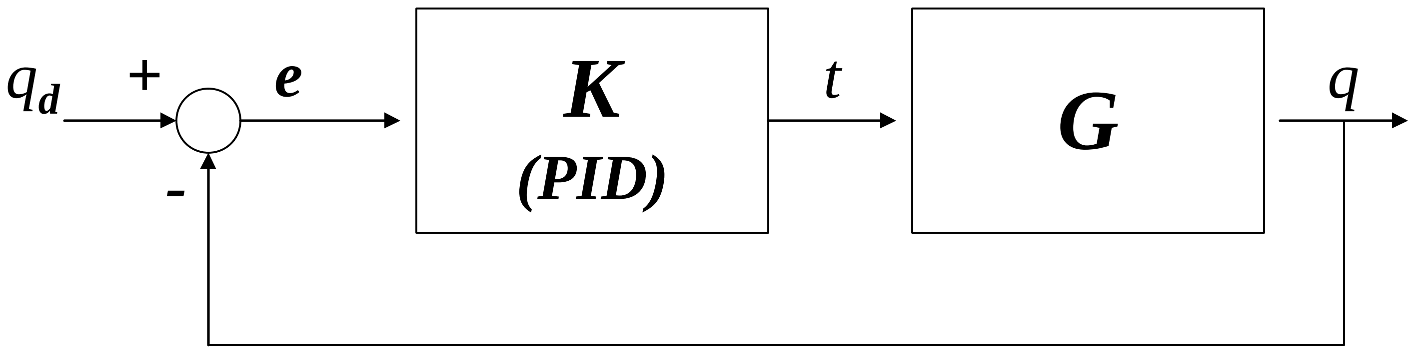
For small K_I, K_D governs the damping and K_P governs the undamped natural frequency w_o

For $K_I > 0$, DC loop gain is infinite, therefore, zero steady state error.

Gain Tuning

- **Intuition:**
  - P gain increases speed of response but also increases overshoot
  - D gain reduces overshoot but decreases speed of response
  - I gain reduces steady state error but can reduce speed of response and lead to instability
- **Strategy:**
  - Tune PD gain until desired transient response is obtained
  - Increase I gain until convergence to steady state is satisfactory.
  - Retune PD gains (increase) if necessary.

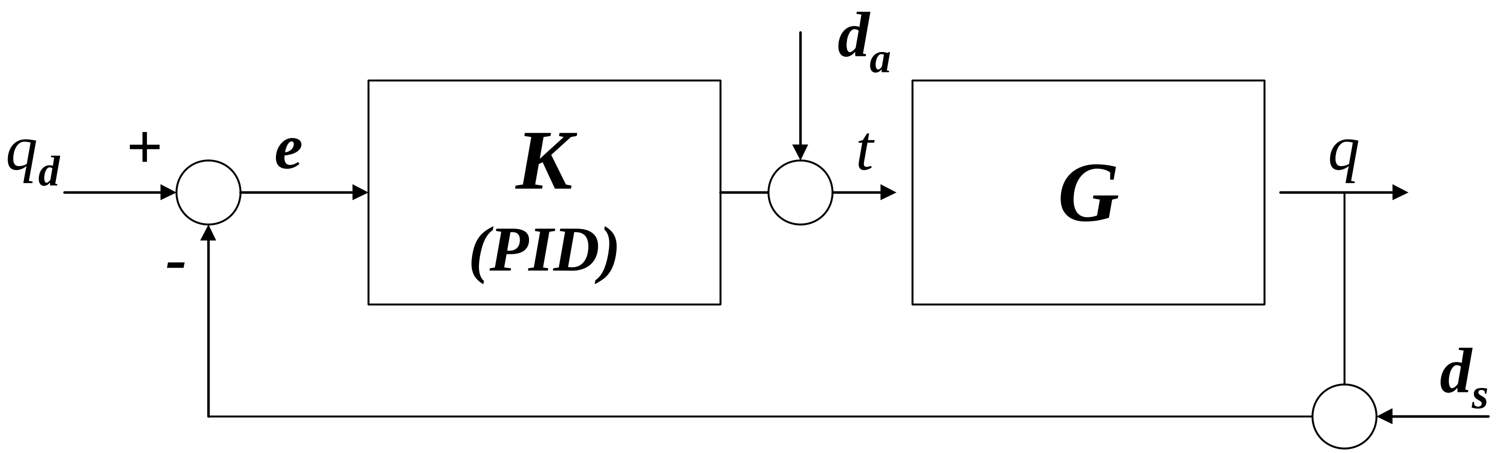
Frequency Domain Considerations



Adjust PID gains to achieve

- **good tracking over the desired bandwidth (transfer function from q_d to q is close to 0dB)**

Disturbance Rejection



Adjust controller (and possibly add more filtering in *K*, but must be careful to preserve stability and dynamical response) so the frequency gain is small over the disturbance frequency.

Next Week

- **More on control design.**

Tomorrow at 6pm in CII 2037

- **Group 1: 6pm, Group 2: 6:15pm, Group 3: 6:30pm, Group 4: 6:45pm, Group 5: 7:00pm, Group 6:7:15pm, Group 7: 7:30pm.**
- **Prepare to discuss the progress of your project.**