Today (10/26/01)

- Today
  - Walk through the lead filter design exercise 10
  - Nyquist plot
  - Ref. 6.3
- Reading Assignment: 6.5

Lead Design Procedure

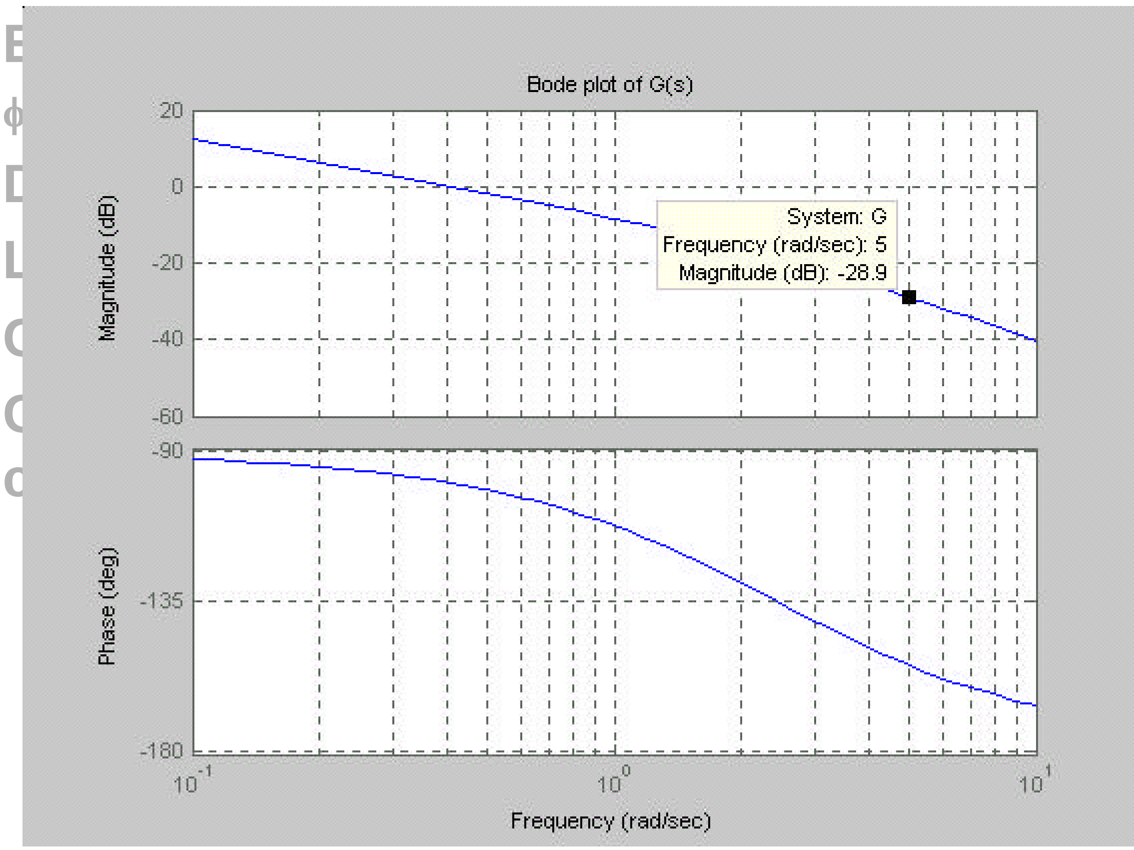
Given *G(s)*:

- **Determine open loop gain *K* to meet low freq gain requirement (*KG(0)*) and/or bandwidth requirement (BW *KG(s)* about ½ of desired closed loop BW). Gain crossover freq=ω_{cg}.**
- **Evaluate PM of *KG(s)*. Determine extra phase lead needed, set it to ϕ_{max} .**
- **Determine α. Find the new gain crossover freq ω_{cg1}** $KG(j\omega_{cg1}) = \left(\sqrt{\alpha}\right)_{\mathrm{dB}}$
- **Let ω_{max}= ω_{cg1} and solve for *T.***
- **Check PM, BW of $G(s)K_{lead}(s)$ and iterate if necessary.**
- **Check all other specifications, and iterate design; add more lead compensators if necessary.**

Lead Design Procedure

Given $G(s)=1/(s^2+as)$:

- **Determine open loop gain K to meet** low freq gain requirement ($KG(0)$) and/or **bandwidth requirement (BW $KG(s)$ about ½ of desired closed loop BW). Gain crossover freq=ω_{cg}.**

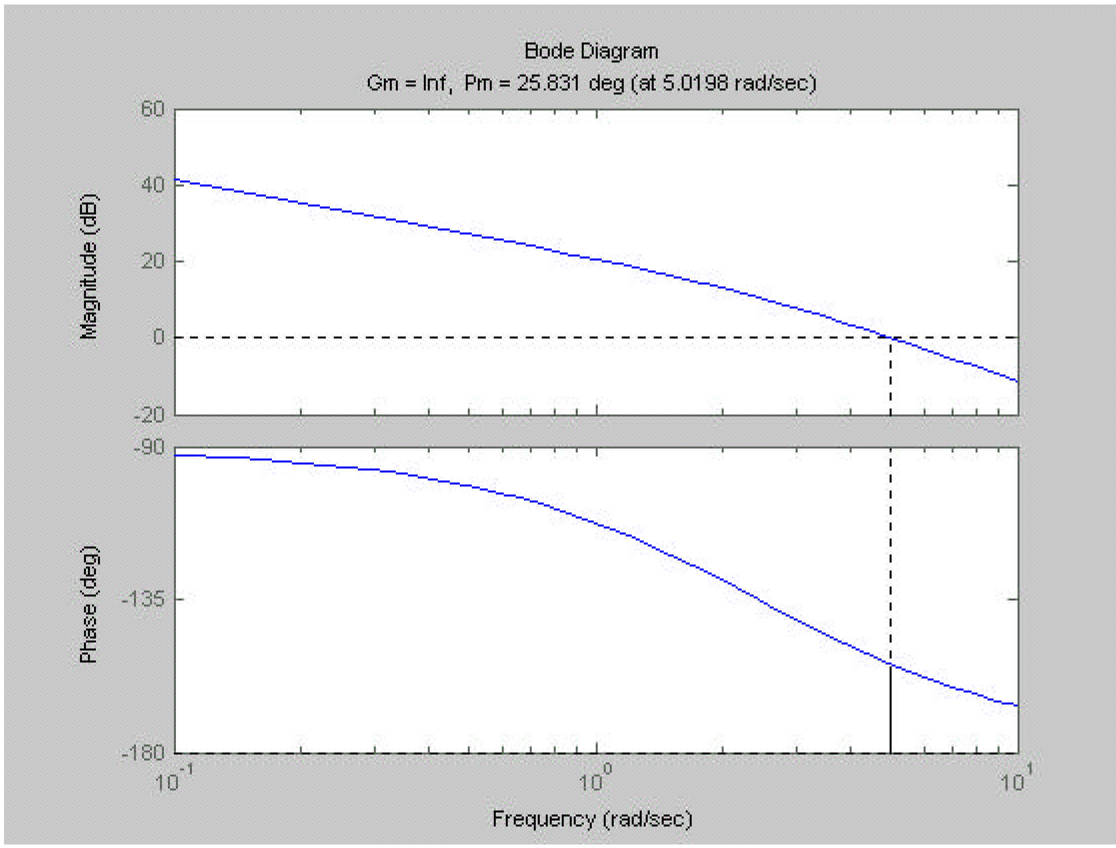


- E … e lead needed, set it to ϕ
- D … eq v
- L
- C … cessary.
- C … sign; add more lead c

Choose K=28.9dB=28

Lead Design Procedure

- **Evaluate PM of *KG(s)*. Determine extra phase lead needed, set it to ϕ_{max} .**



Target PM=60deg, so we need 35deg from lead filter. Add 5 deg extra pad.

$$f_{max} = 40\text{deg} = .684\text{rad}$$

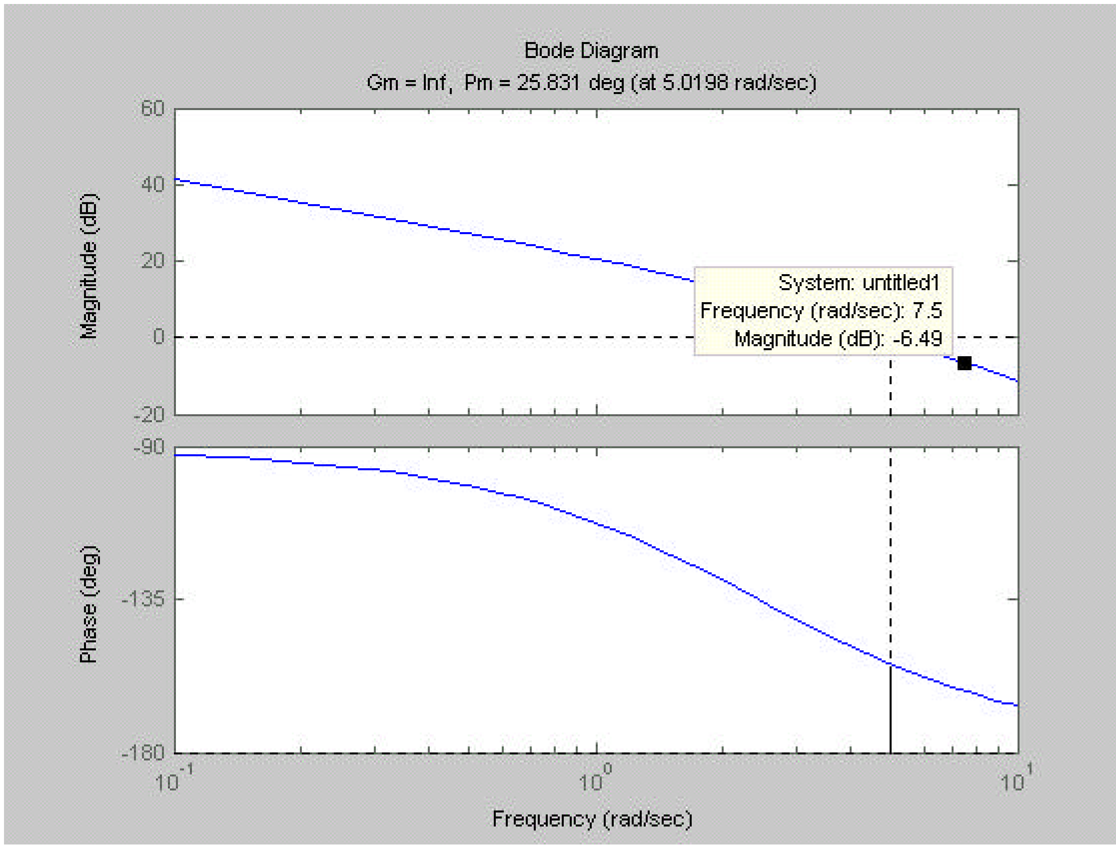
$$a = .225$$

extra gain from lead filter at w_{max}

$$= \left(\sqrt{a}\right)_{dB} = 6.47\text{dB}$$

Lead Design Procedure

- **Determine α. Find the new gain crossover freq w_{cg1}**



New crossover freq @ 7.5rad/sec.

Substitute $w_{\max} = 7.5\text{rad/sec}$ into

$$T = \frac{1}{w_{\max}\sqrt{a}}$$

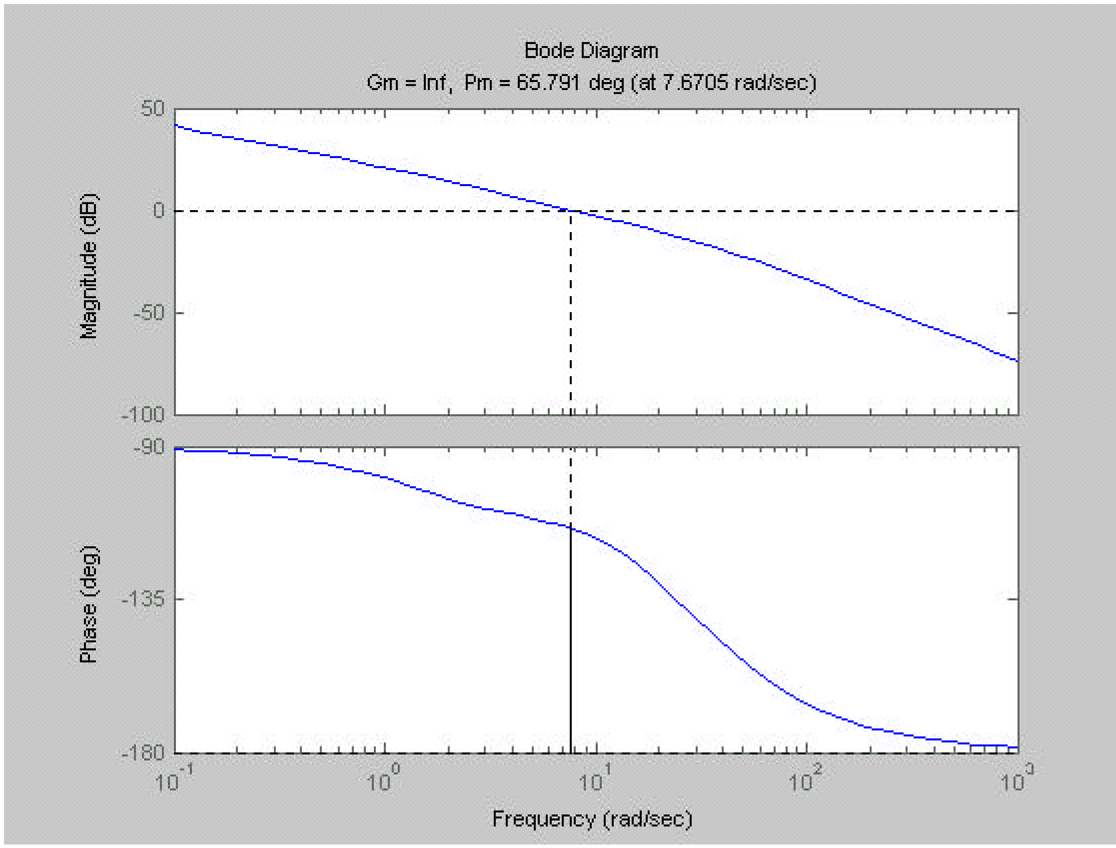
we get $T = .28$

Overall lead filter:

$$K_{lead}(s) = 124.2\,\frac{s+3.56}{s+15.79}$$

Lead Design Procedure

- Check PM, BW of $G(s)K_{lead}(s)$ and iterate if necessary.



Not quite meeting the spec, so iterate!

Change $\phi_{max} = 50^o$ and

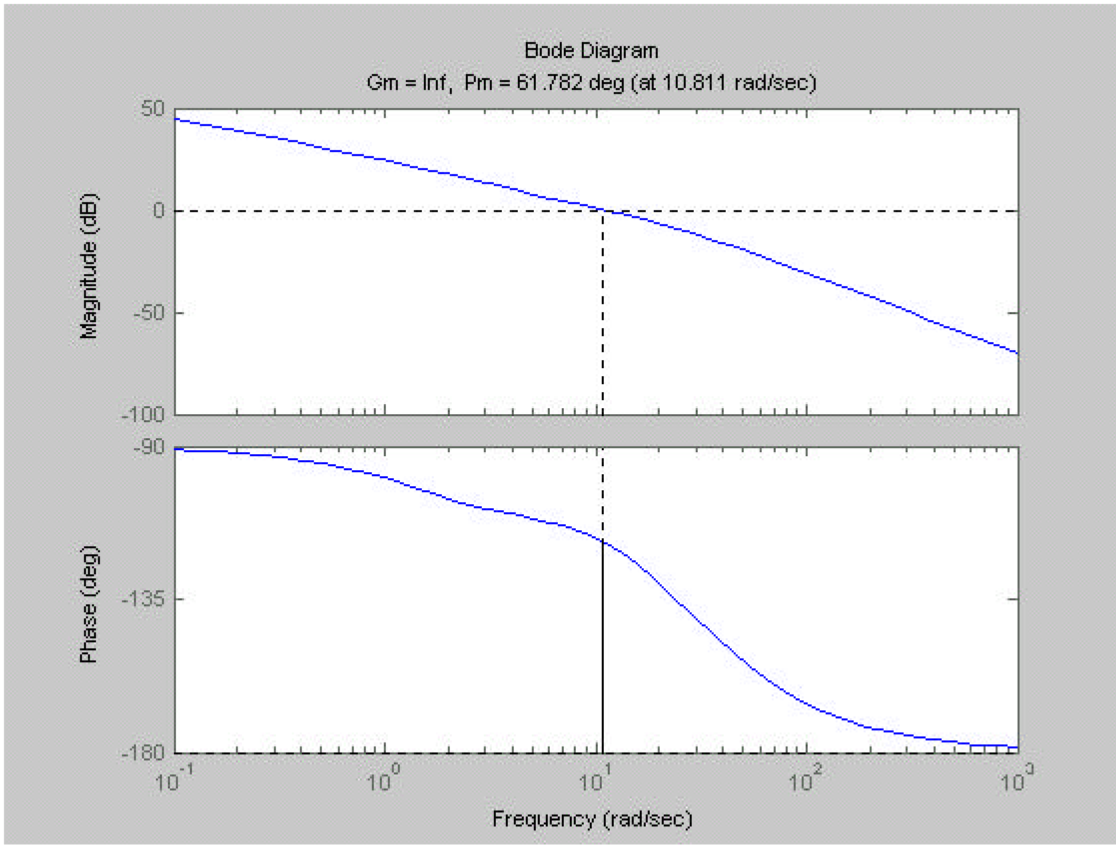
$\omega_{max} = 10\text{rad/sec}$

Overall lead filter:

$$K_{lead}(s) = 202.4\frac{s+3.72}{s+26.89}$$

Lead Design Procedure

- **Check PM, BW of $G(s)K_{lead}(s)$ and iterate if necessary.**

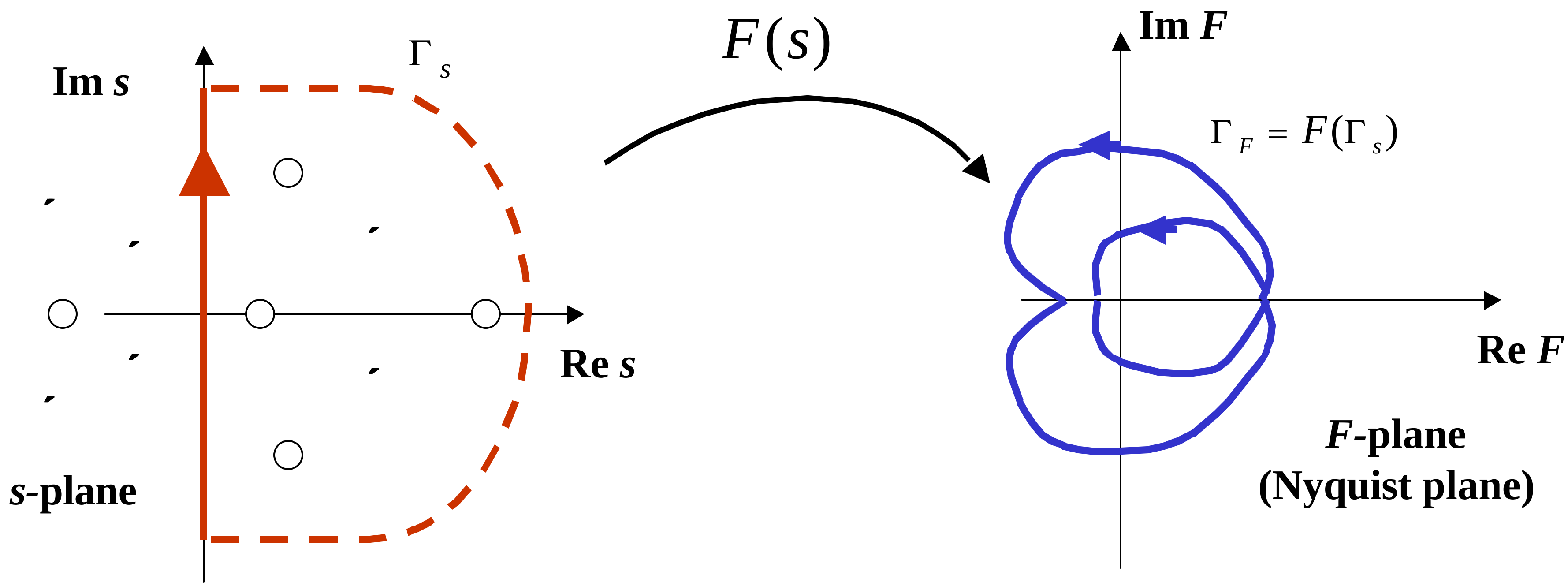


More PM than we need, so increase the gain a bit.

Overall lead filter:

$$K_{lead}(s) = 303.7\frac{s+3.72}{s+26.89}$$

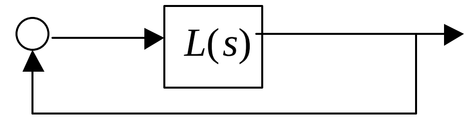
Principle of Argument



Let *Z*=# of zeros of *F(s)* enclosed by G, *P*=# of poles of *F(s)* enclosed by G.

Then # of counterclockwise (CCW) encirclement of origin of *F*-plane by *F*(G) = *P*-*Z*

Nyquist Stability Criterion



Choose:

- **G encloses the entire right half complex plane**
- ***F(s)=1+L(s)***

Then

- **Z = # of unstable zeros of $1+L(s)$**

  = 0 if the closed loop system is stable
- **P = # of unstable poles of $1+L(s)$**

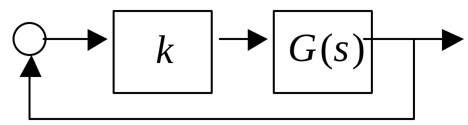
  = # of open loop unstable poles (# of unstable poles in $L(s)$)

From the Principle of Argument,

closed loop system is stable if and only if

# of CCW encirclement of the origin in the Nyquist plane by G_{1+L} = # of open loop unstable poles

Nyquist Stability Criterion



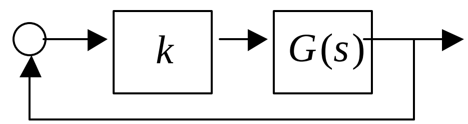
If *L(s)* = *k G(s)*, then

closed loop system is stable if and only if

# of CCW encirclement of (-*1/k,0*) in the Nyquist plane by G_G = # of open loop unstable poles

Example

$$G(s)=\frac{s+2}{s^2-1}$$



$$G(jw)=\frac{jw+2}{-w^2-1}$$

$$\operatorname{Re}G(jw)=-\frac{2}{1+w^2}$$

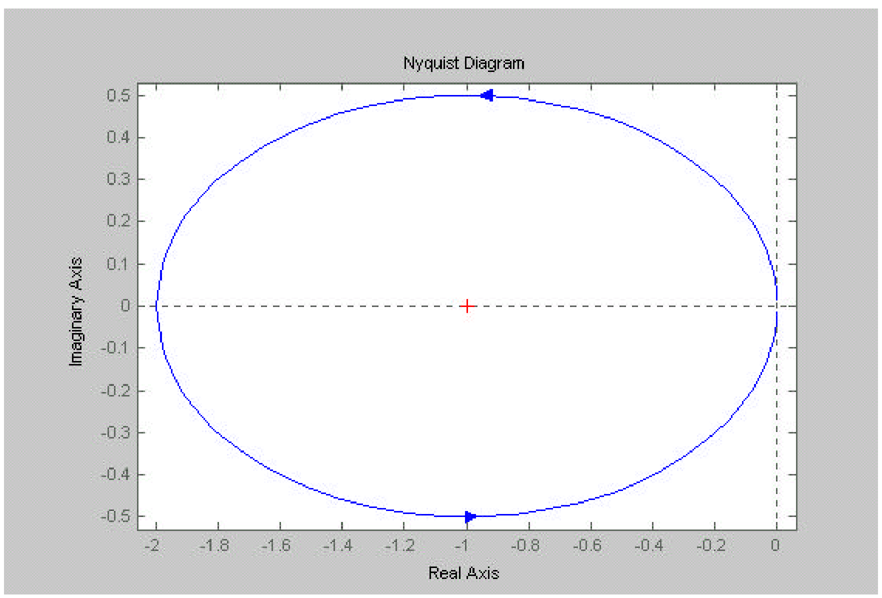
$$\operatorname{Im}G(jw)=-\frac{w}{1+w^2}$$

When $w=-\infty, G(jw)=0.$

When $w<0, \operatorname{Im}G(jw)>0$

When $w \nearrow 0, \operatorname{Re}G(jw)\to -2$

$w>0$ case is the mirror image



Closed loop is stable

$$\Leftrightarrow \quad -2<-\frac{1}{k}<0 \quad \Leftrightarrow \quad k>\frac{1}{2}$$

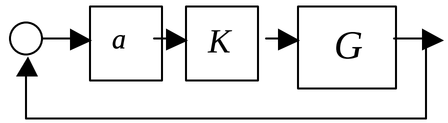
Check:

$$\text{num}(1+kG(s))=s^2-1+k(s+2)$$

MATLAB command:

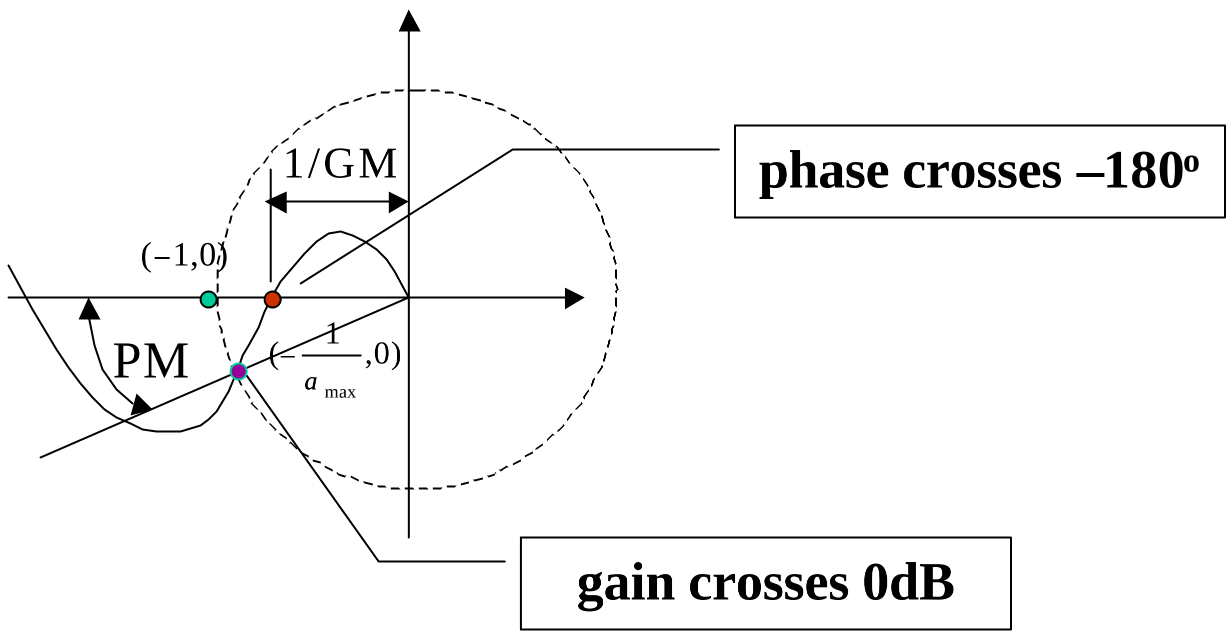
```
nyquist(G);
```

Gain/Phase Margins



Nominal value of a=1.

To evaluate the stability margin (how much a can vary), check the Nyquist plot of *GK:*

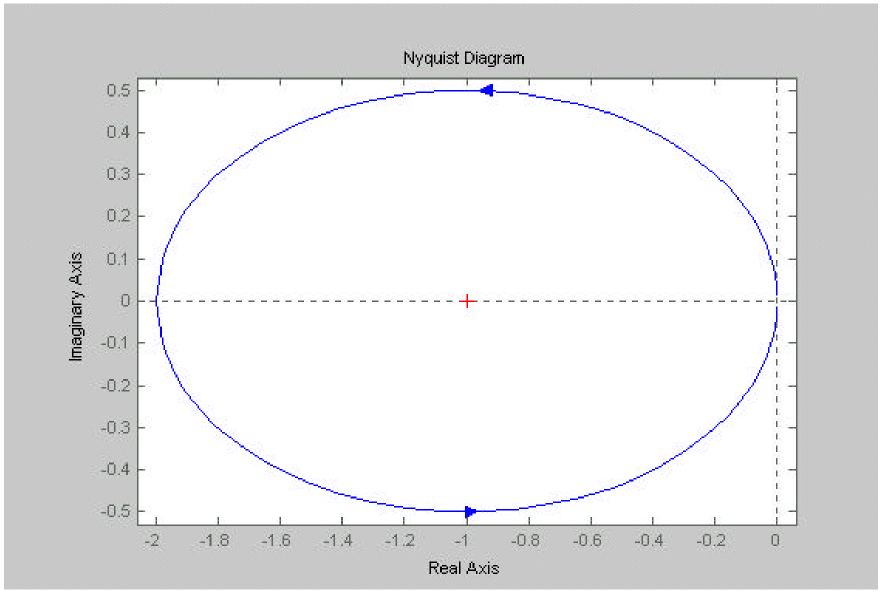


Example

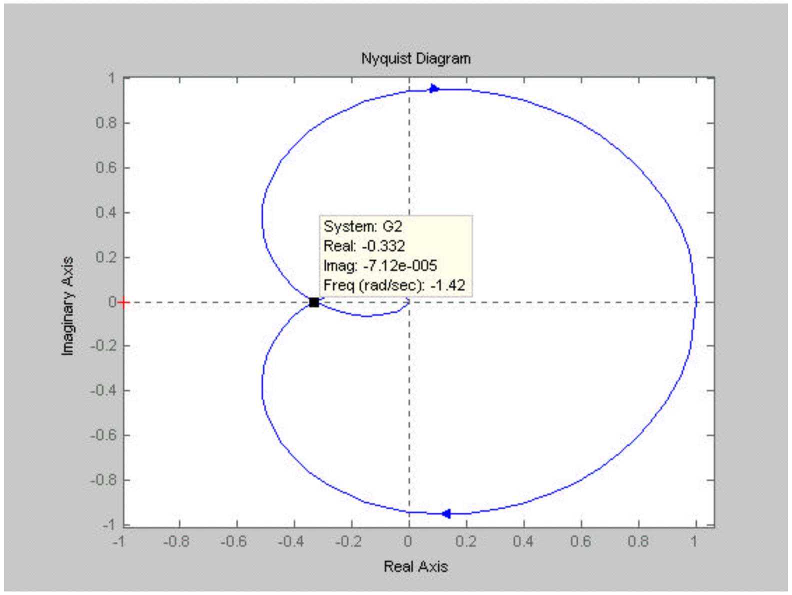
$$G(s) = \frac{s+2}{s^2 - 1}$$

$$G(s) = \frac{1}{s^3 + 2s^2 + 2s + 1}$$

Nominal feedback gain *k*=1



max gain: 1/2=0.5=-6dB GM

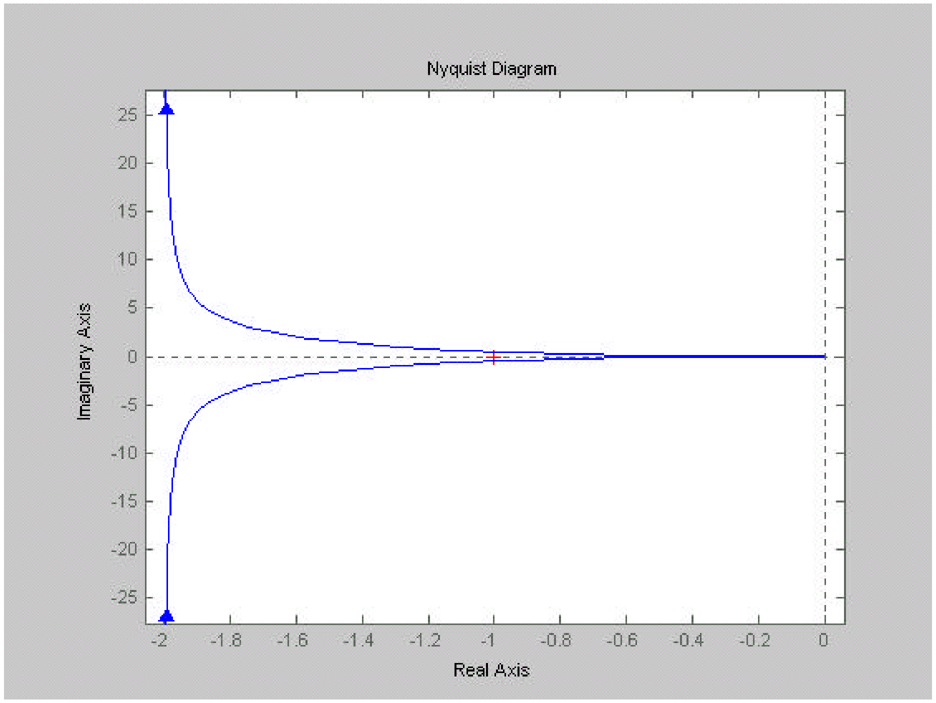


max gain: 1/.332=3.01=9.6dB GM

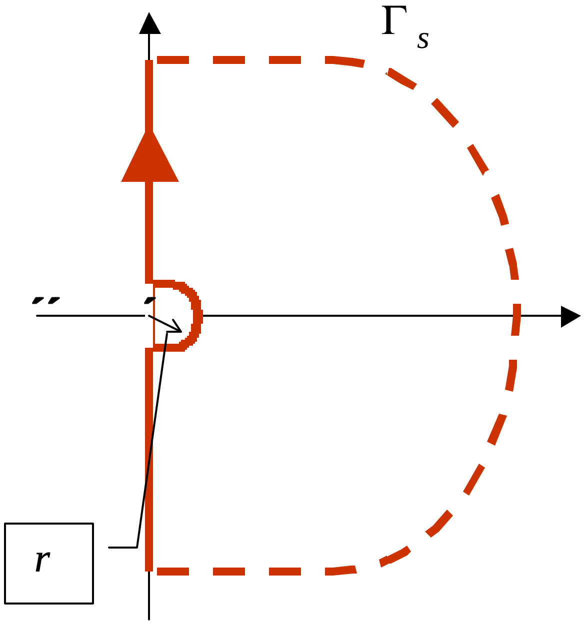
Zero on the imaginary axis

$$G(s) = \frac{1}{s(s+1)^2}$$

$G(jw) \to \infty$ as $w \to 0$ so Nyquist plot becomes unbounded.



which direction does the plot go at infinity?



$r \to 0$

$$G(s) = \frac{1}{s(s+1)^2} \approx \frac{1}{s} \text{ for } s \approx 0$$

$$G(r e^{jf}) \approx r^{-1} e^{-jf}$$

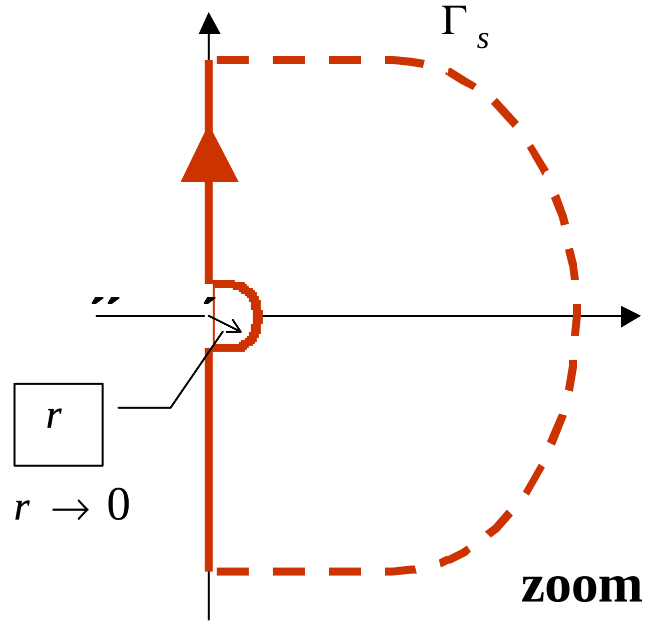
Zero on the imaginary axis

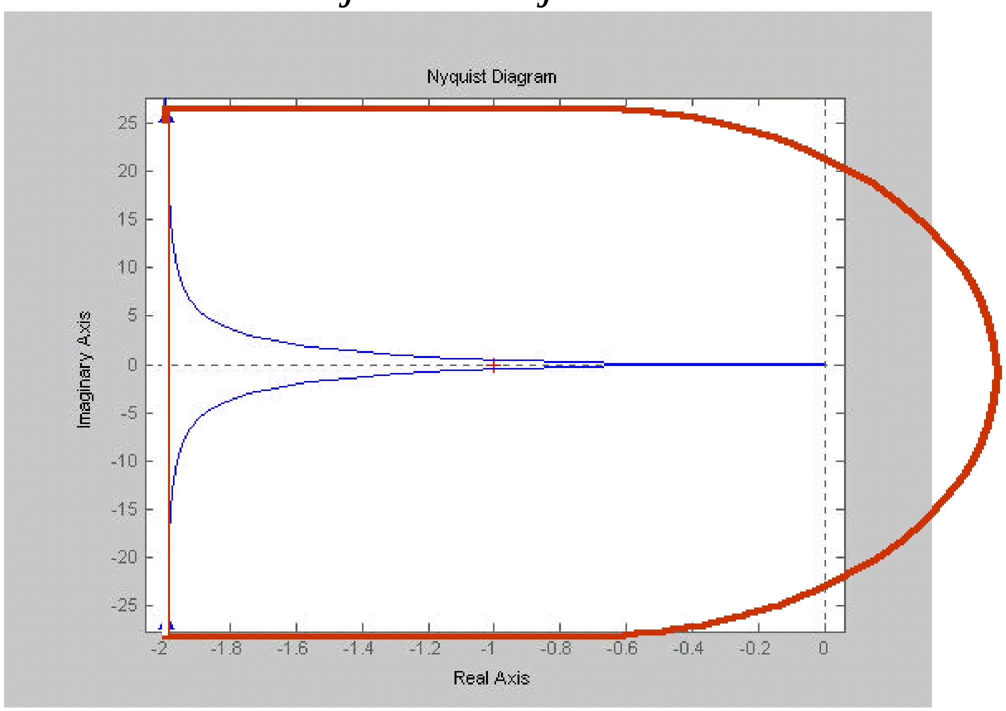
$$G(s) = \frac{1}{s(s+1)^2} \approx \frac{1}{s} \text{ for } s \approx 0$$

$$G(r\, e^{jf}) \approx r^{-1} e^{-jf}$$

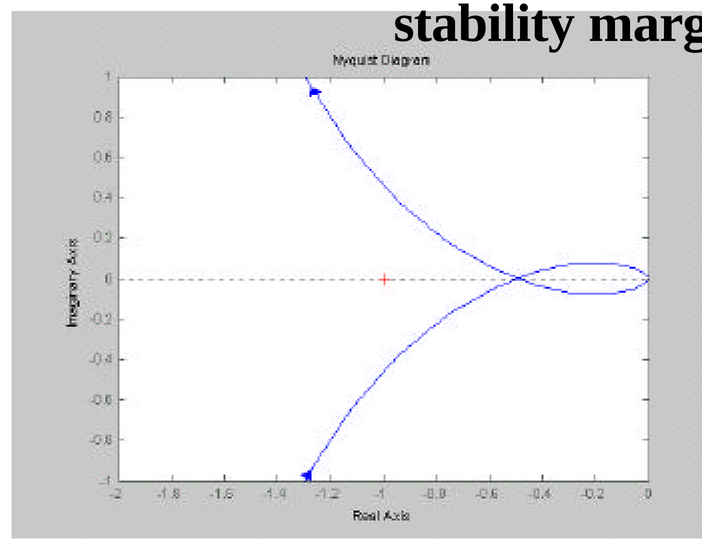
$$f : -\frac{p}{2} \rightarrow 0 \rightarrow \frac{p}{2}$$

$$e^{-jf} : j \rightarrow 1 \rightarrow -j$$





zoom in for stability margins



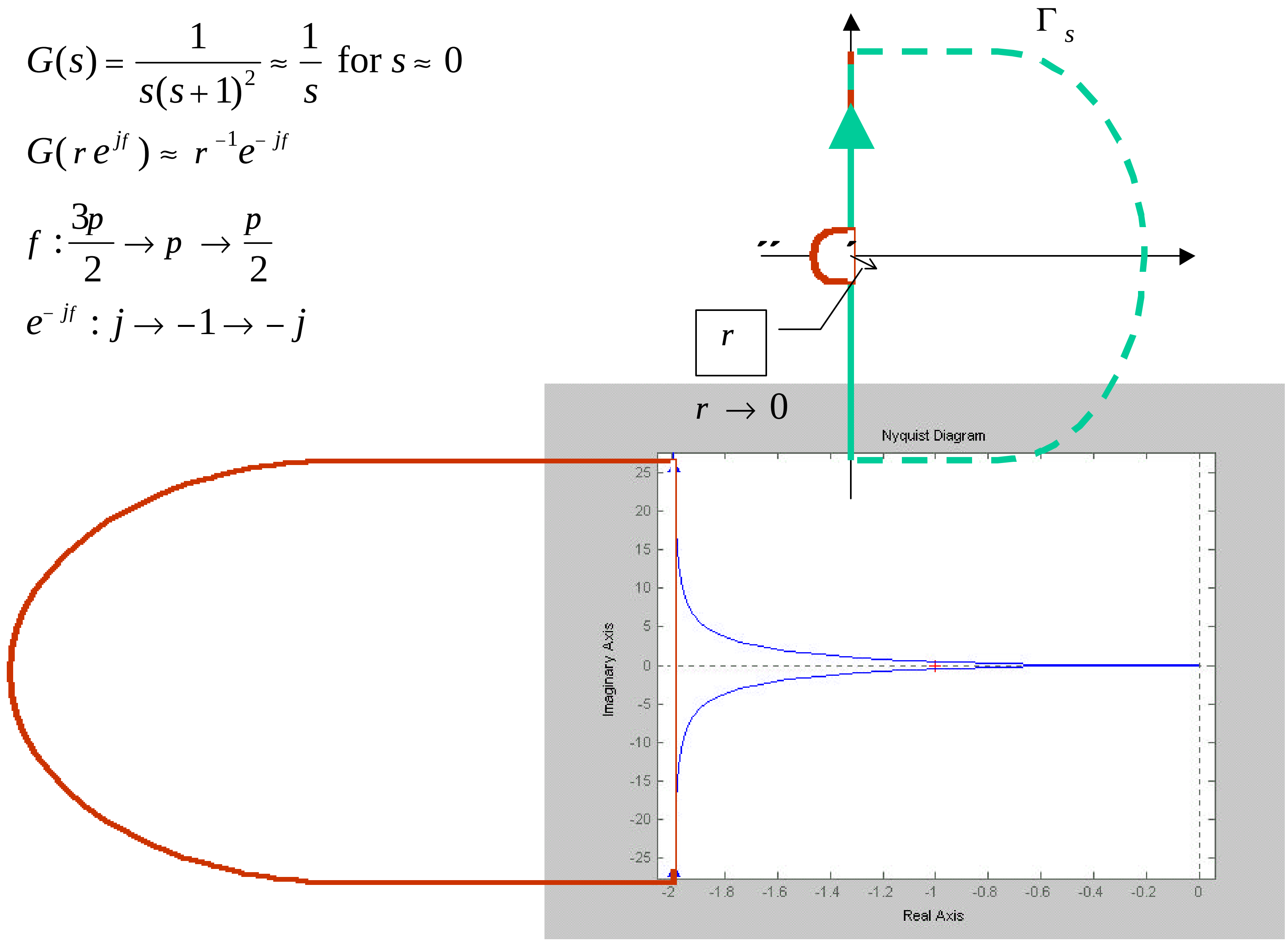
Zero on the imaginary axis

$$G(s)=\frac{1}{s(s+1)^2}\approx\frac{1}{s}\ \text{for } s\approx 0$$

$$G(r\,e^{jf})\approx r^{-1}e^{-jf}$$

$$f:\frac{3p}{2}\to p\to\frac{p}{2}$$

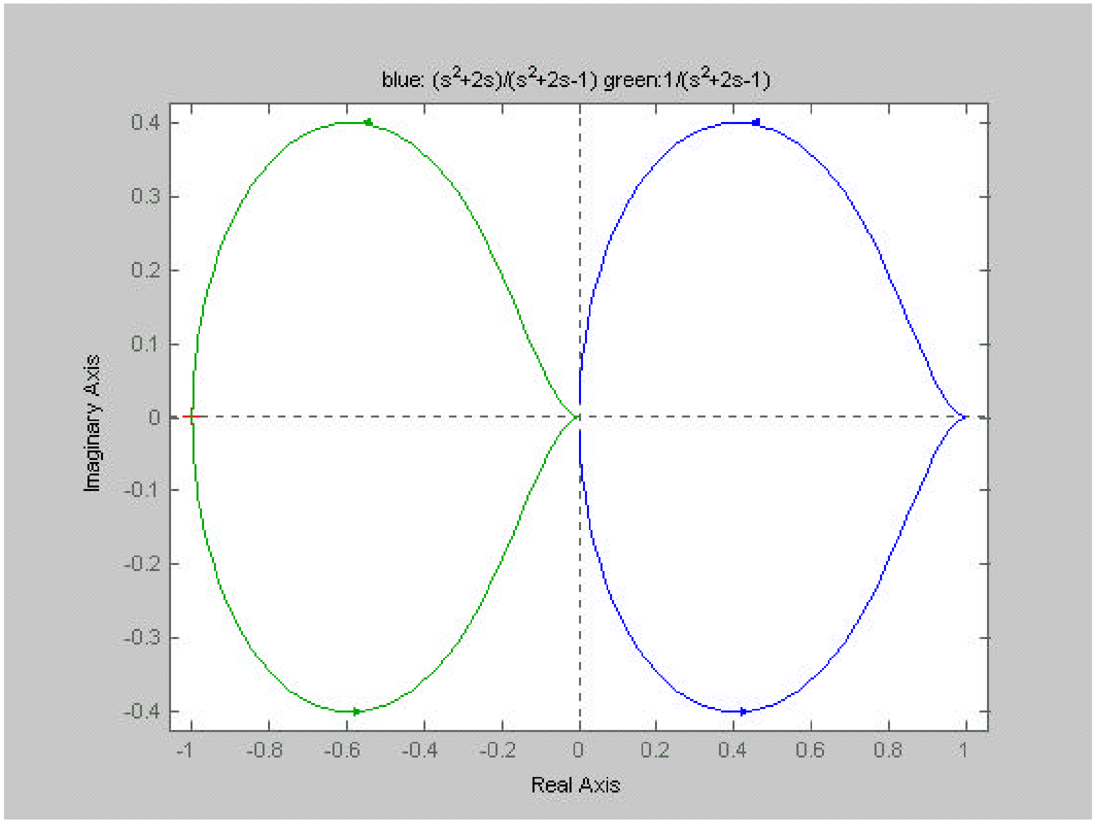
$$e^{-jf}: j\to -1\to -j$$



Systems with Z=P

For strictly proper loop gain *L(s)* (more poles than zeros), we only need to evaluate the Nyquist plot for *s=j*ω since *L(s)* $\to$ *0* as $\omega \to \pm\infty$. When *L(s)* has the same # of poles and zeros, this is no longer true. In this case, write *L(s)* as a constant + strictly proper transfer function. Plot the Nyquist plot of *L(s)* and shift the plot by the constant.

$$G(s) = \frac{s(s+2)}{(s^2+2s-1)} = 1 + \frac{1}{s^2+2s-1}$$



Exercise 11

Consider the lead filter $K_{lead}(s)=300(s+4)/(s+30)$ applied to the plant $G(s)=1/s(s+a)$:

- **Do the Nyquist plot of $G(s)$ by hand and compare with the MATLAB generated Nyquist plot.**
- **Use the MATLAB generated Nyquist plot for $G(s)K_{lead}(s)$ to evaluate the gain and phase margins. Compare with the results from using the Bode plot.**