Mechanical Design

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Control System Design

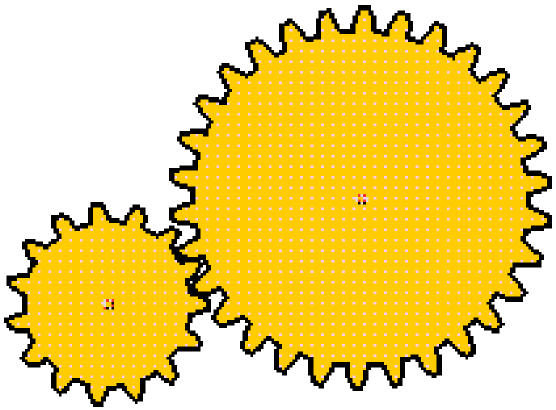
February 26, 2003

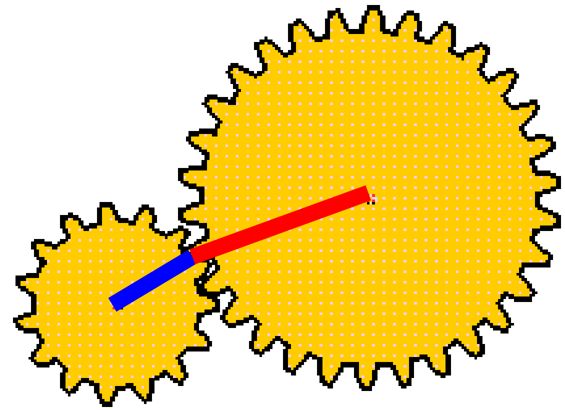
Lecture Contents

- Miscellaneous Components
  - Gears and Belts
  - Flexible Couplings
  - Bearings
  - Fixing Components to Shafts
- Dynamics
- Stress and Strain Analysis
- Heat Transfer Analysis
- PDE &The Finite Element Method
- Design Studies

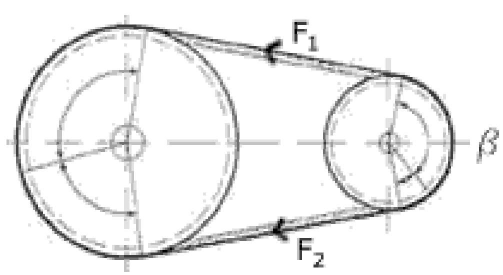
Movies
  - Flexible Mirror Positioning (Robust Design)
  - Pendubot Link Design (Mass & Stress Optimized)
  - Robot Link Design (Controllability Optimized)

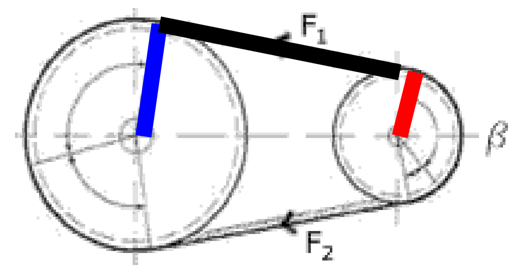
Gears and Belts





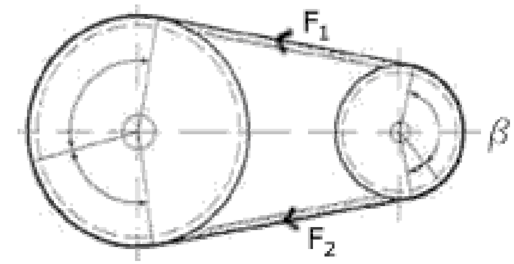
Instantaneously, gears and pulleys act like levers





Gears and Belts (2)

At steady state velocity or ignoring inertia



FBD ⇔ **Thermodynamic** ⇔ **Kinematic**

(F and M balance) — (conservation of energy) — (velocity analysis)

$$N = \frac{r_1}{r_2} = \frac{\tau_1}{\tau_2}$$

$$\tau_1 = N\tau_2$$

$$Power = \tau\dot{\theta}$$

$$\tau_1\dot{\theta}_1 = \tau_2\dot{\theta}_2$$

$$N = \frac{\dot{\theta}_2}{\dot{\theta}_1} = \frac{\tau_1}{\tau_2}$$

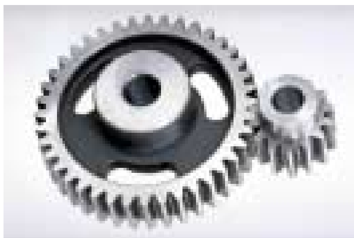
$$N = \frac{r_1}{r_2} = \frac{\dot{\theta}_2}{\dot{\theta}_1}$$

$$\dot{\theta}_2 = N\dot{\theta}_1$$

Conservative System

(Power in = Power out)

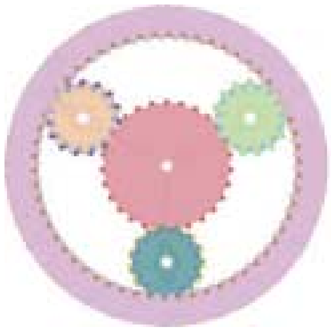
Gearheads - Spur



- Inexpensive
- Efficient
- Low torque
- Some backlash

Motor and Gearhead often combined

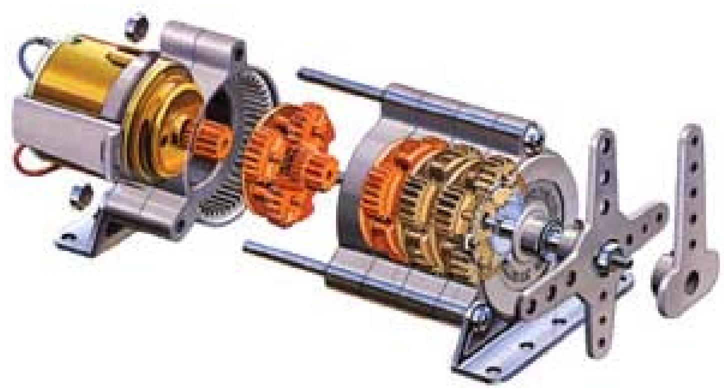
Gearheads - Planetary



Large reduction in a small and lightweight assembly

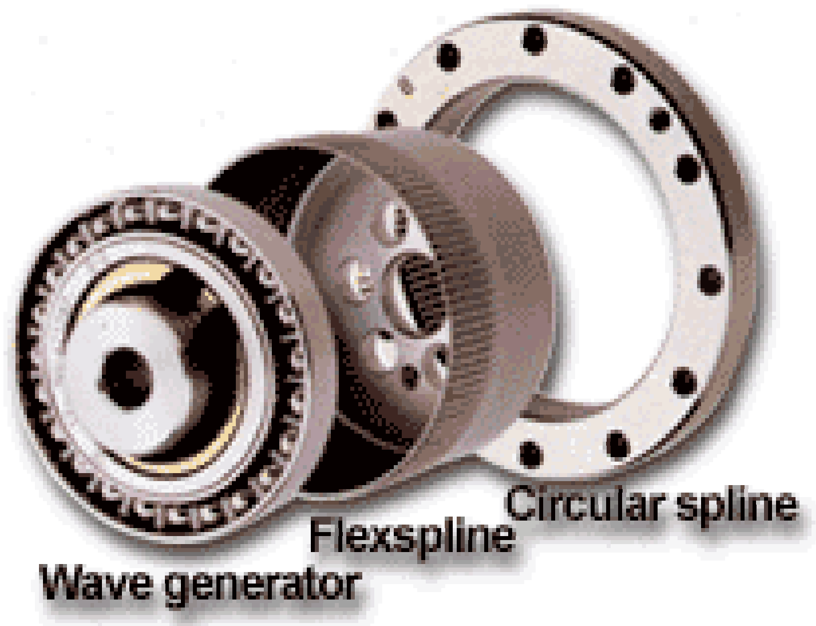
Large torque capacity

Considerable backlash



Identical stages can be stacked together

Gearheads - Harmonic



Flexspline has 2 fewer teeth than circular spline

One tooth advances every rotation

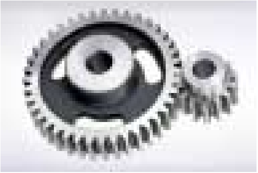
Huge reduction in small and lightweight assembly

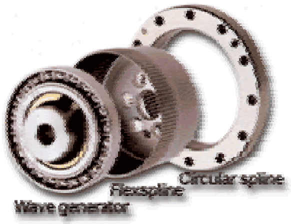
Almost no backlash

Aerospace and robotics applications

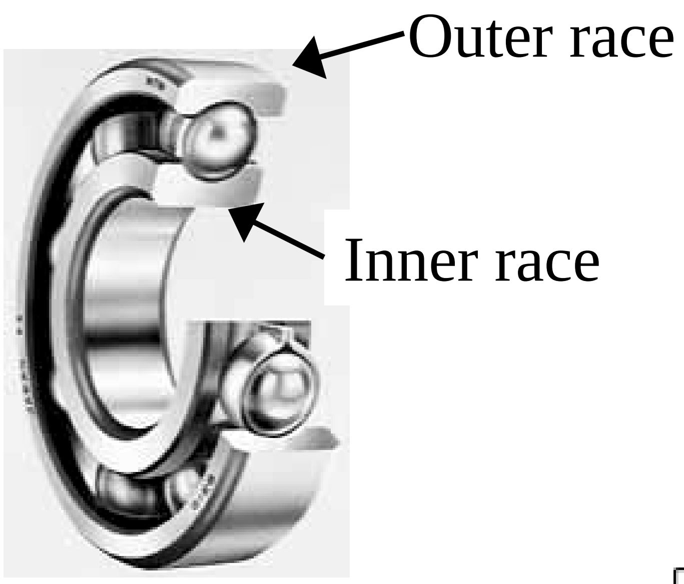
Gearhead Comparison

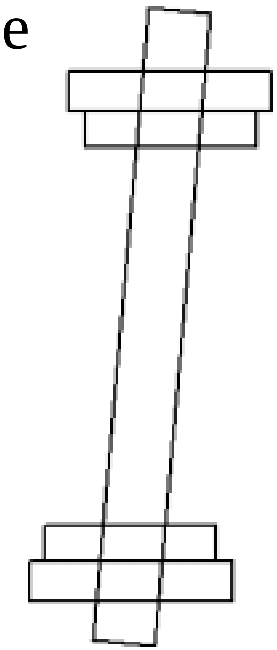
Type	**Torque**	**Efficiency**	**Backlash**	**Cost**
Spur	Low	High	Low	Low
Planetary	High	Low	High	Med
Harmonic	Med	Med	Low	High





Bearings (1)





Bearings allow for slight misalignment

Bearings are usually press fit to the shaft or housing.

Correct bearing hole diameter is critical.

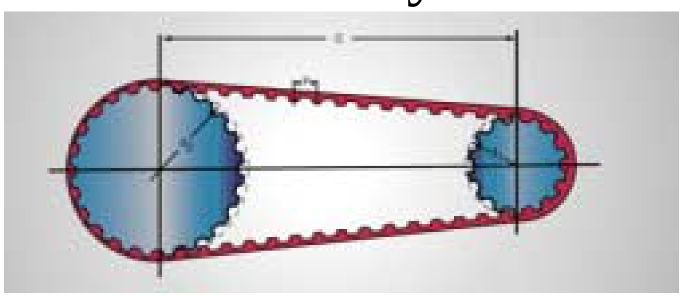
Heat expansion can be used to assist assembly.

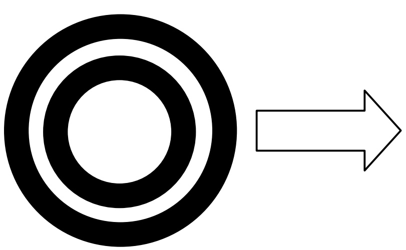
Bearings (2)

Load conditions determine mounting scheme to prevent bearing "creep". Creep is when the bearing race rolls on the mating surface.

Press fit bearing race (inner or outer) that experiences cyclic loading

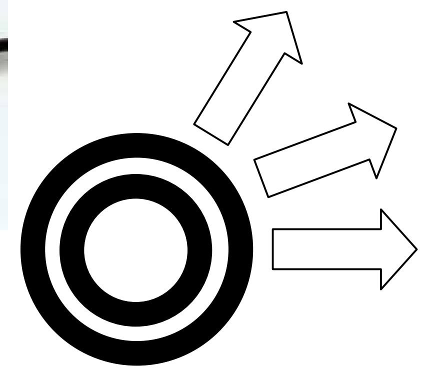
Belt Drive System





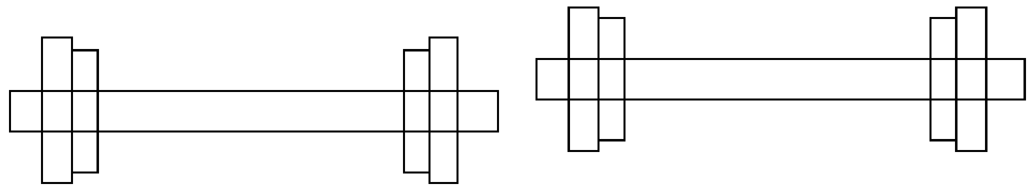
Shaft "sees" varying load so press fit shaft

Centrifuge with unbalanced load

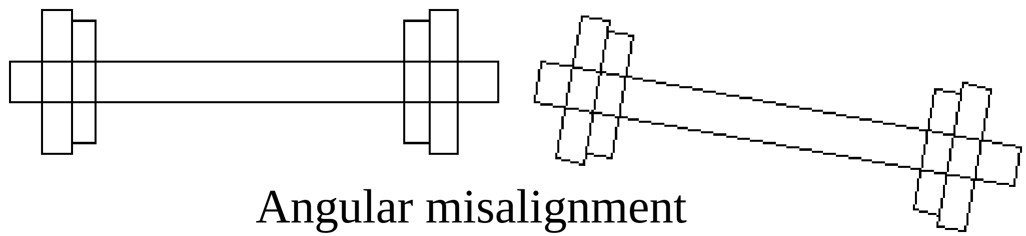


Housing "sees" varying load so press fit housing

Flexible Couplings



Parallel offset misalignment



Angular misalignment

- Coupling is used to transmit torque between two shafts.
- Coupling is rigid in torsion and flexible in bending.
- Without flexible coupling, there would be excessive loads on the shafts and bearings.
- Without flexible coupling, bearings would fail prematurely and performance would suffer.

Fixing Components to Shafts

Method	**Torque**	**Stress**	**Cost**	
Setscrew	Low	High	Low	unreliable
Clamp	High	Low	Med	
Pin	Low	High	Low	
Key	Med	High	Med	
Spline	High	Low	High	

Dynamics

Ordinary differential equation:

$$M(\theta)\ddot{\theta} + C(\theta, \dot{\theta})\dot{\theta} + B(\dot{\theta}) + G(\theta) = \tau$$

Solution:

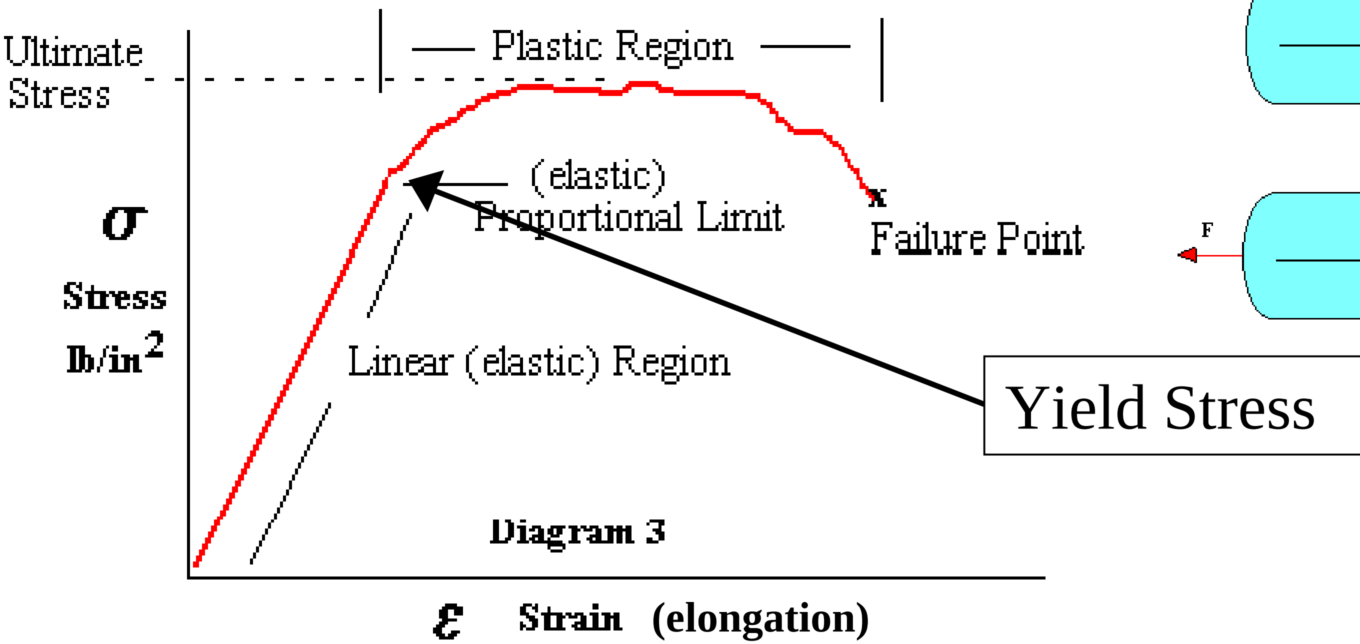
Numerical integration (MATLAB ODE solvers)

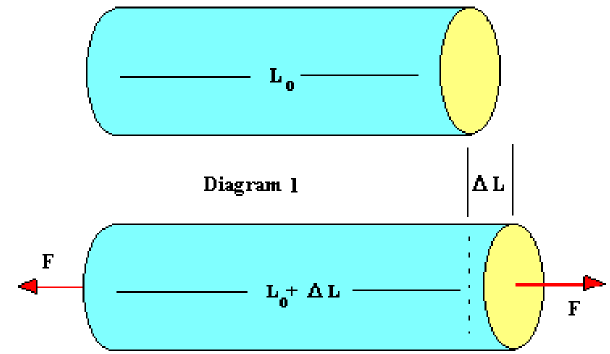
Dynamic forces are exerted on the mechanical components.

$$F = m\ddot{x} \qquad \tau = J\ddot{\theta}$$

Dynamic forces cause deflections and stresses in the structural components.

Stress and Strain (1)





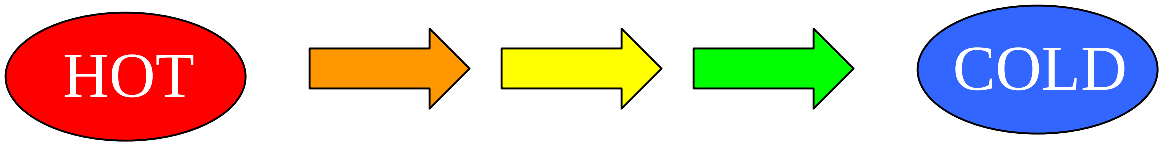
In the linear region, material acts like a spring (F = kx).

Stresses in the components should never exceed the Yield Stress to prevent permanent deformation.

When would we not design to yield strength?

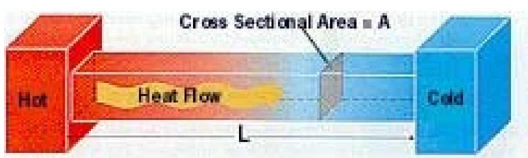
Heat Transfer (1)

Heat transfer is energy transfer due to a temperature difference.



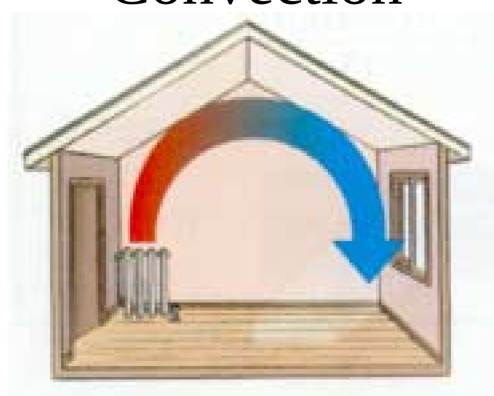
Modes of heat transfer

Conduction



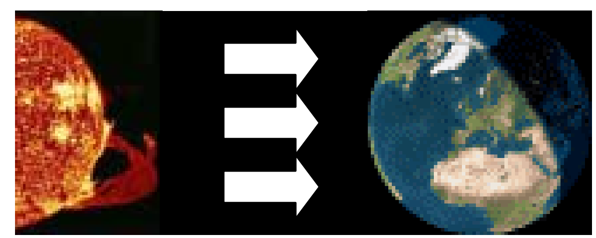
Solid

Convection



Solid Surface to Fluid

Radiation



No Medium

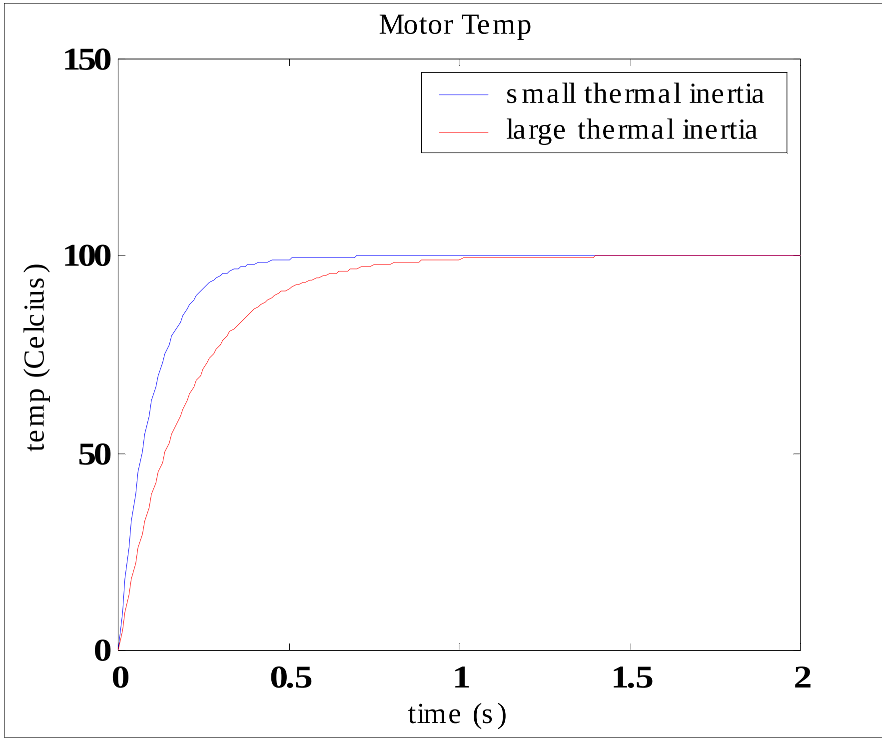
Electromagnetic Waves

Why are we concerned with heat transfer?

Heat Transfer (2)

$$i^2 R$$

Internal heat generation



Why is there a max continuous current for a motor?

Can we exceed this max continuous current?

Partial Differential Equations

Stress and Strain

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} + F_x = 0$$

$$\frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yz}}{\partial z} + F_y = 0$$

$$\frac{\partial \sigma_z}{\partial z} + \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + F_z = 0$$

Equations of Equilibrium

Heat Transfer - Conduction

$$\vec{q}'' = -k\nabla T = -k\left(\vec{i}\,\frac{\partial T}{\partial x} + \vec{j}\,\frac{\partial T}{\partial y} + \vec{k}\,\frac{\partial T}{\partial z}\right)$$

Fourier's Law

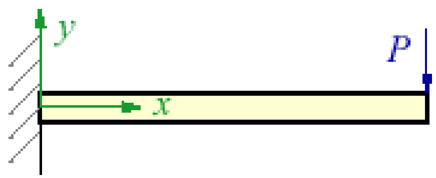
q" = heat flux (energy across unit area per unit time)

T = temperature

Solutions to Partial Differential Equations

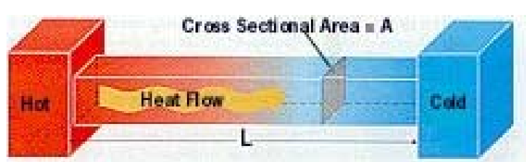
Analytical solutions exist for basic geometry only:

Stress and Strain -Cantilevered beam



$$y = \frac{Px^2}{6EI}(x - 3l)$$

Heat Transfer – 1-D Conduction



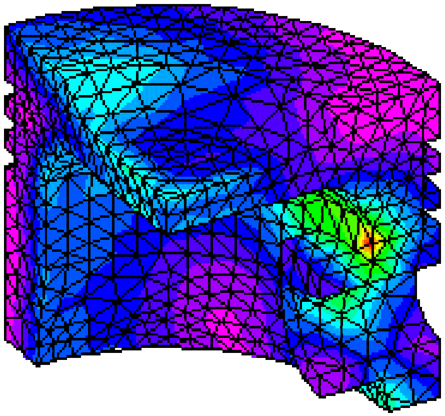
$$T(x) = (T_2 - T_1)\frac{x}{L} + T_1$$

What if the geometry is complex?

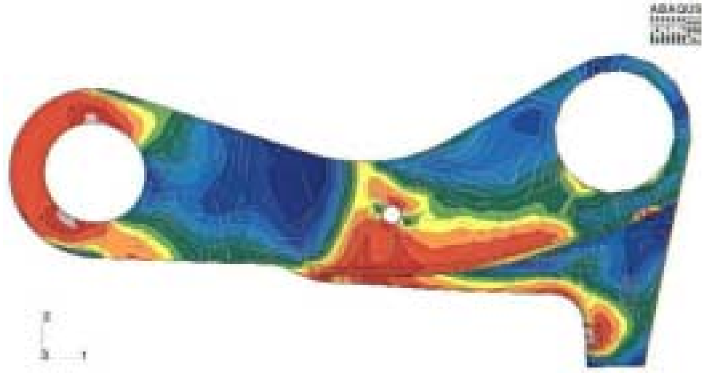
Finite Element Method

Method to find an approximate solution to PDE by discretization.

- Solution converges as mesh is refined (Galerkin Method).
- Very time consuming.
- Computation time vs. modeling accuracy.
- What resolution mesh is required?



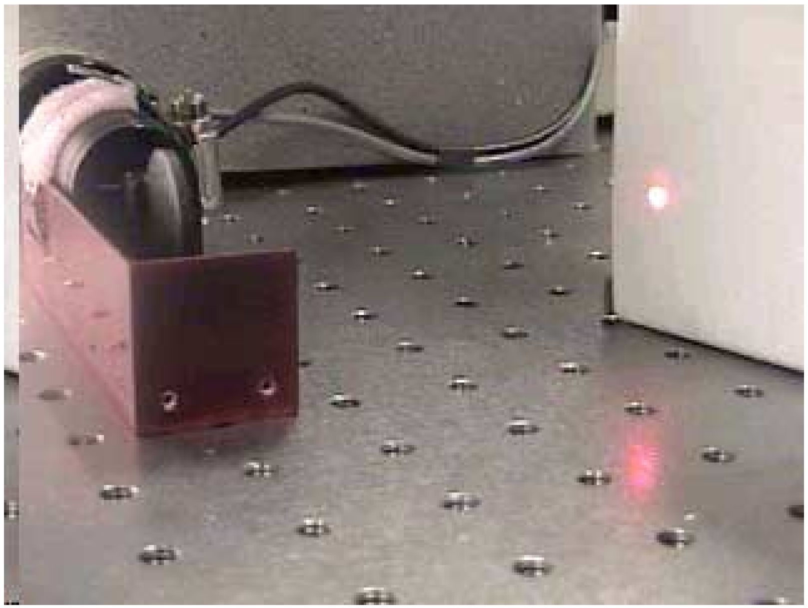
Stress in piston



Hook Stress

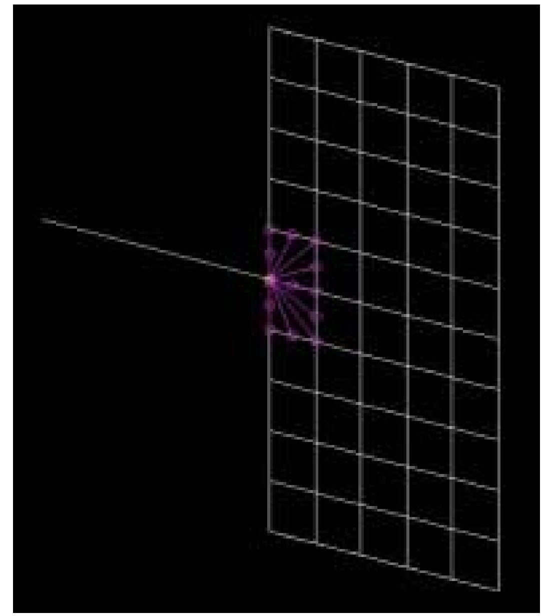
Design Study – Flexible Mirror (1)

Rapid positioning of laser beam



PD control with actuator saturation

Finite Element Mesh



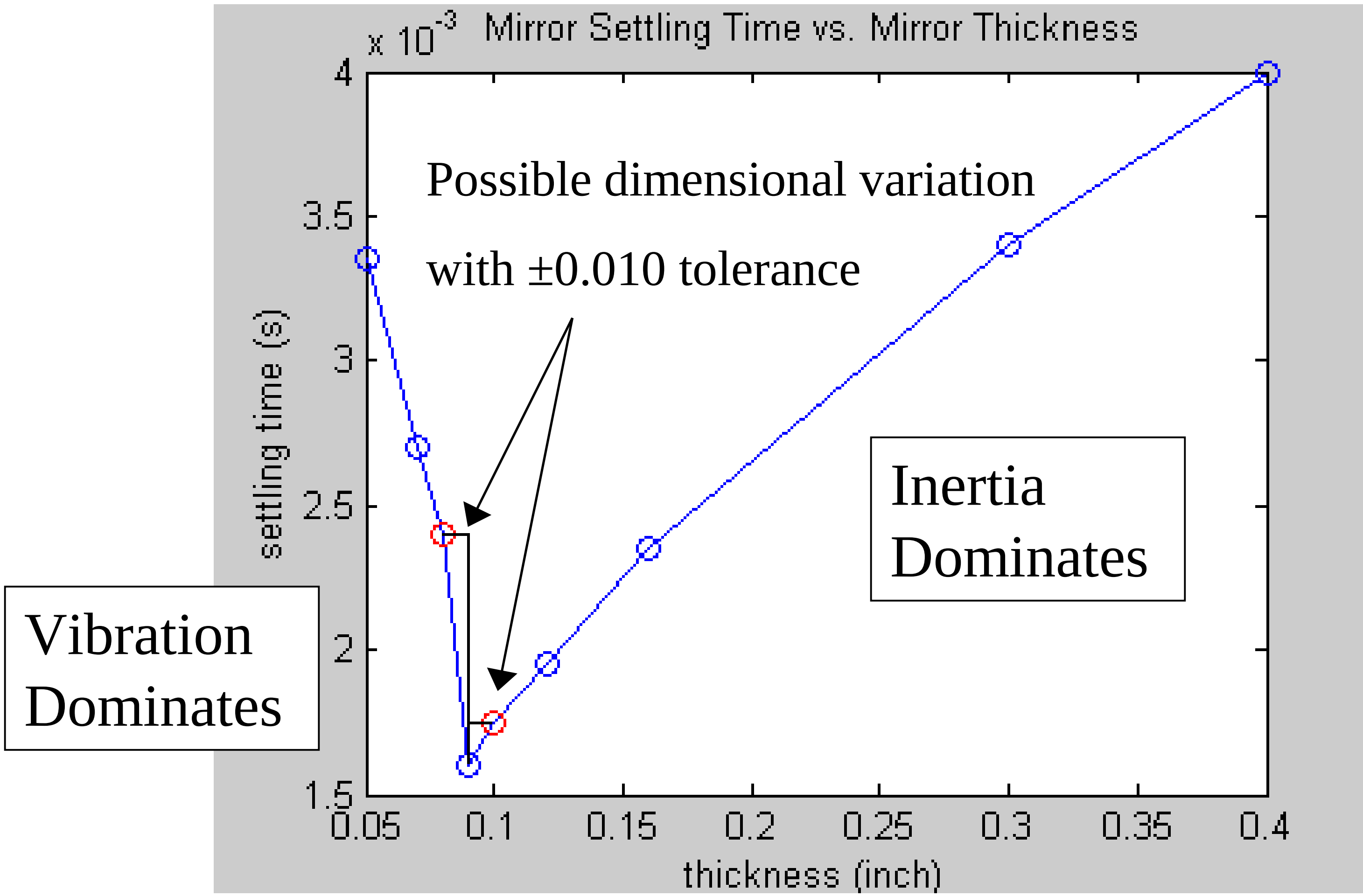
ODE+PDE

High inertia (thick mirror) increases settling time.

Vibrations (thin mirror) increases settling time.

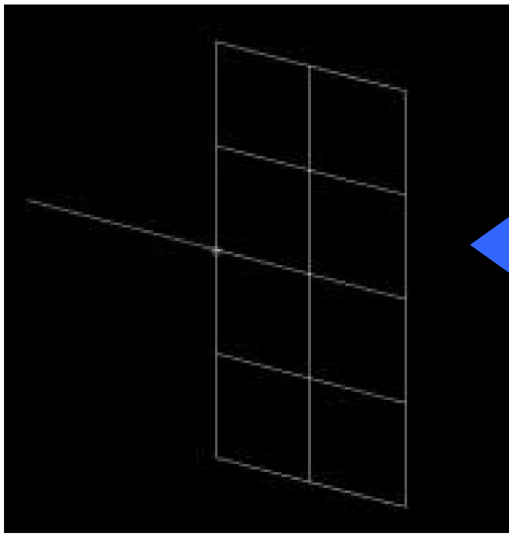
What mirror thickness minimizes settling time?

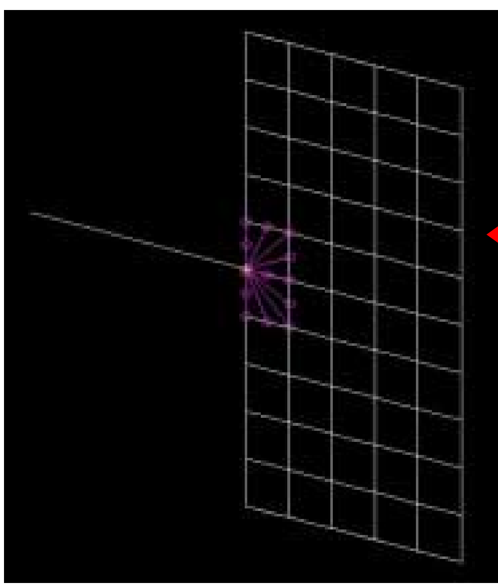
Design Study – Flexible Mirror (2)

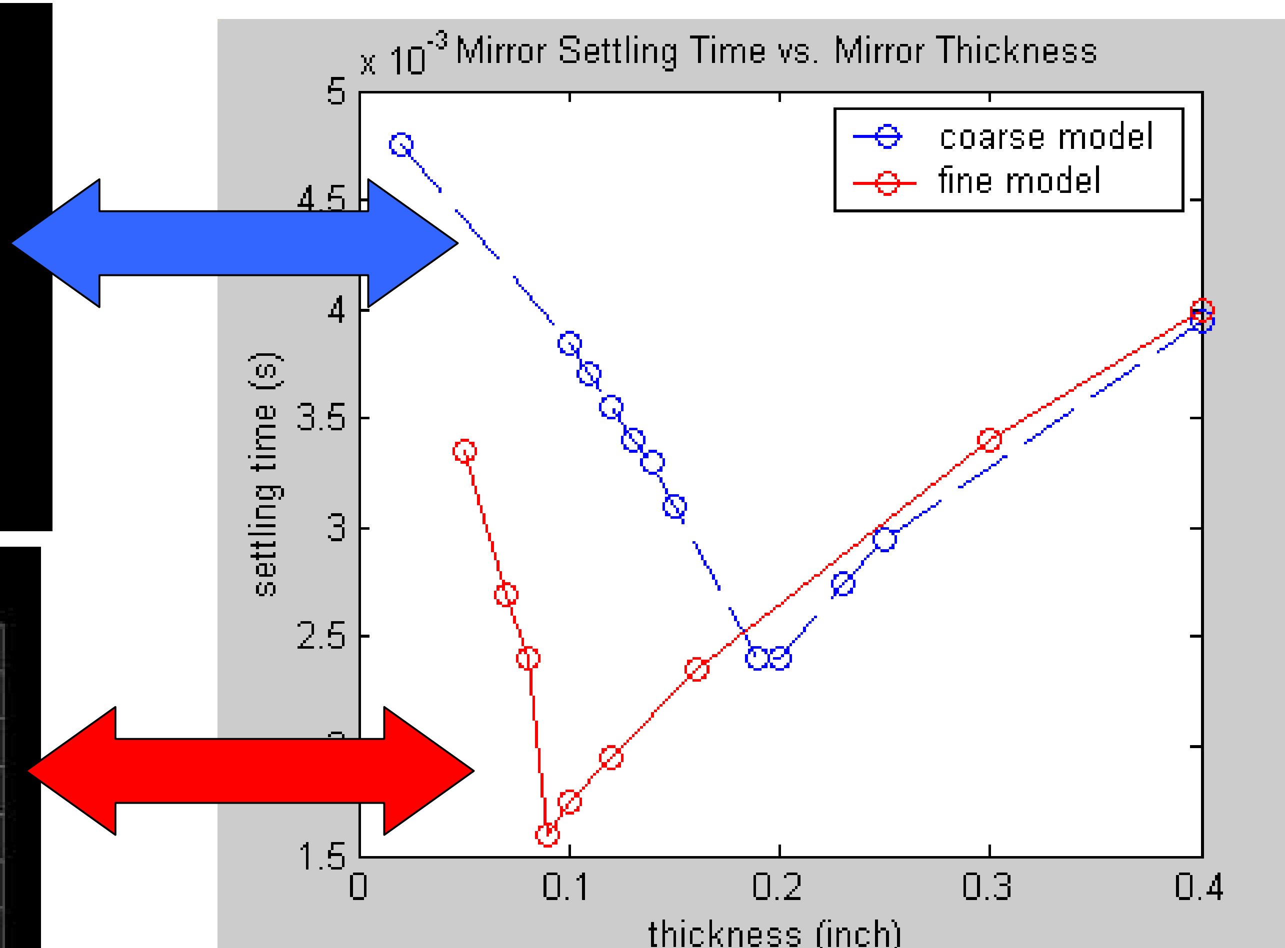


How would you specify the thickness of the mirror for high performance that is robust against manufacturing tolerances?

Design Study – Flexible Mirror (3)



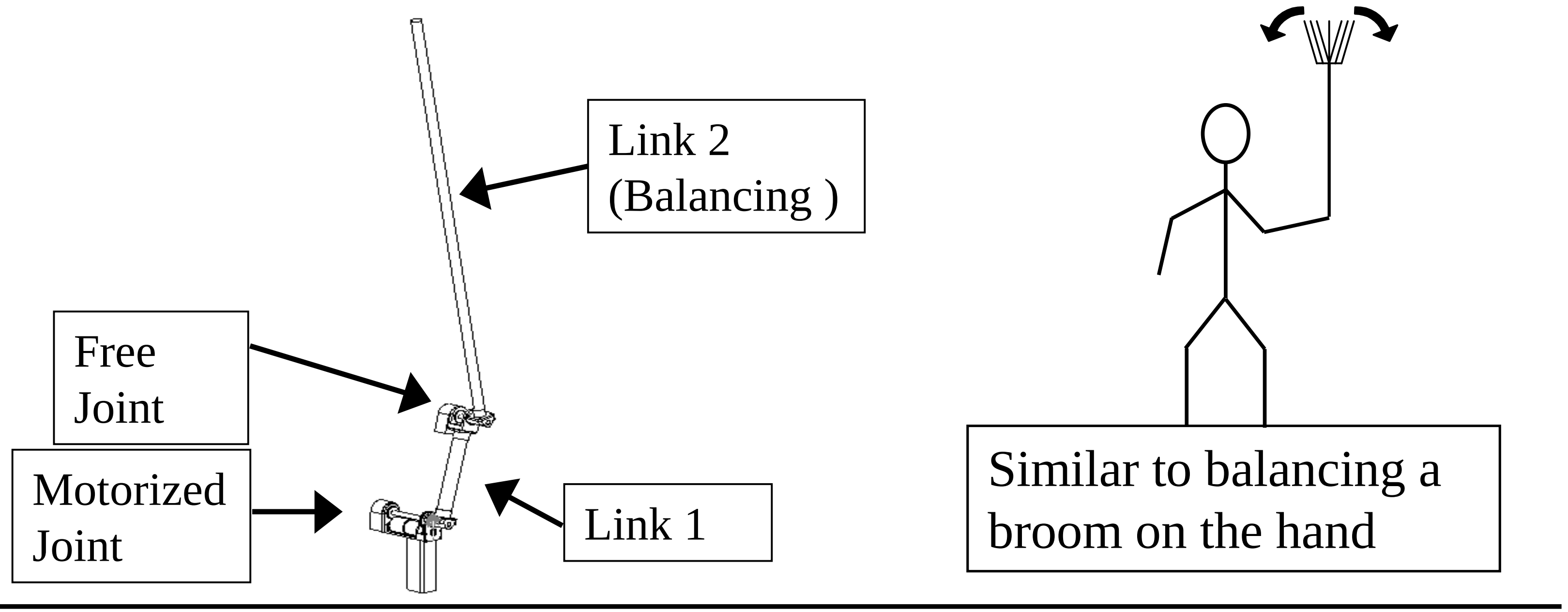




What fidelity model is required?

Computation time vs. accuracy

Design Study – Pendubot Link (1)



* Link 1 is subject to dynamic loading conditions *

GOAL = minimize the mass of link 1 but do not exceed the yield strength of the material

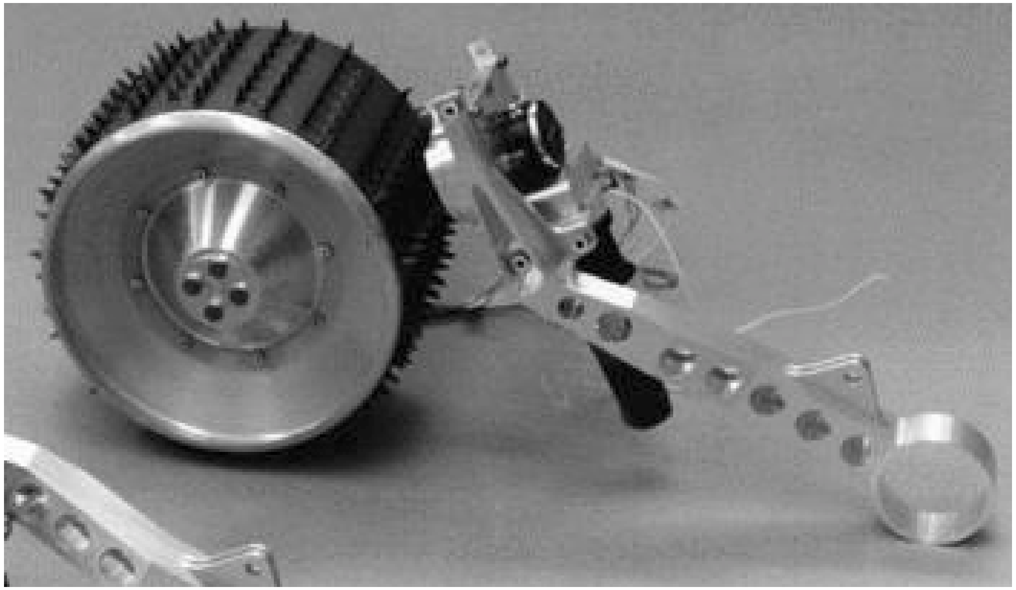
Design Study – Pendubot Link (2)

Pendubot Balancing

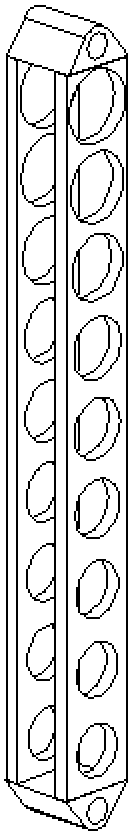
* Lower Link is subject to dynamic loads *

Design Study – Pendubot Link (3)

- Very rough approximation of T-slot hollowing technique
- 9 Design Variables = Dia.of the holes
- Material = Aluminum 6061T-6

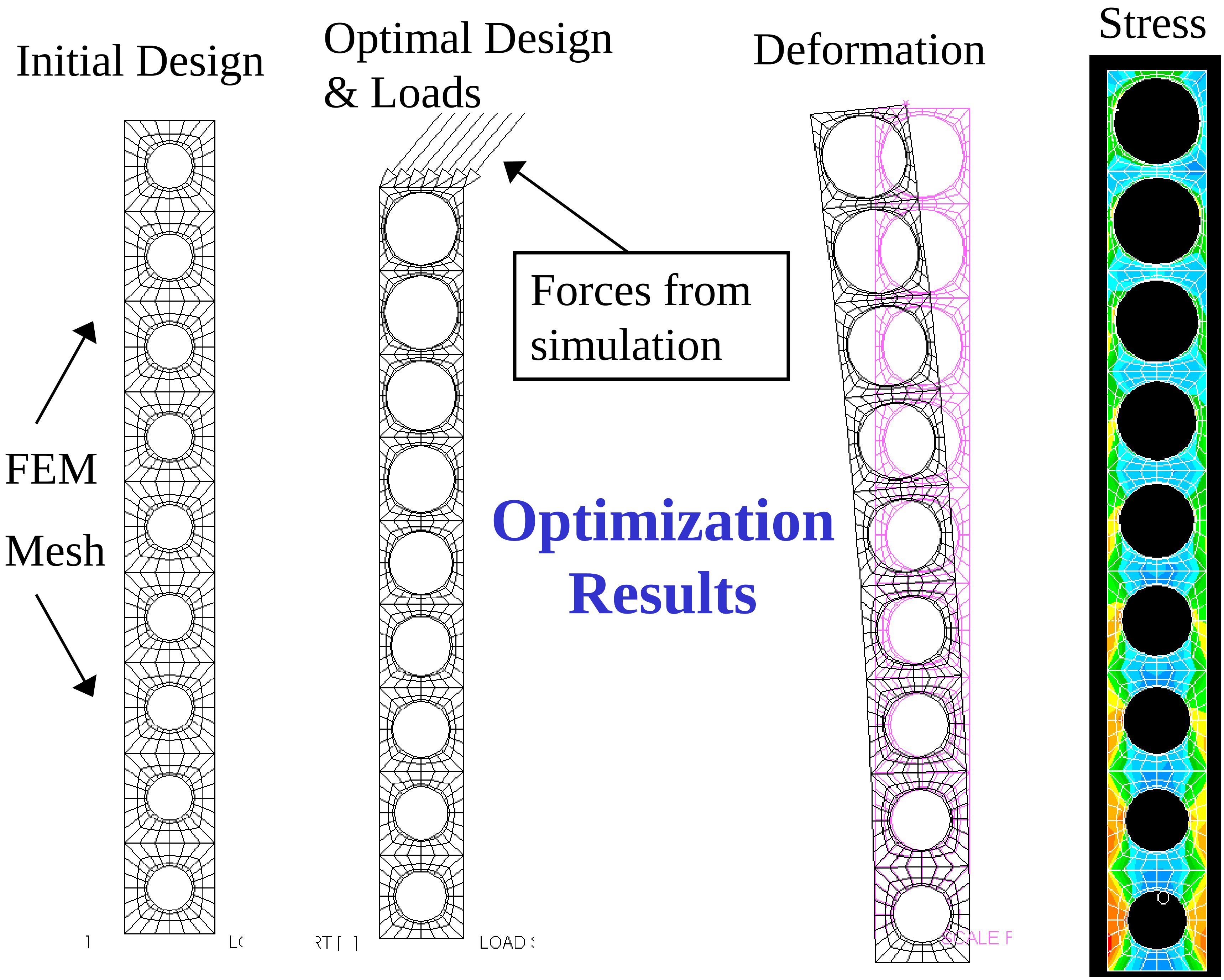


T-slot hollowing

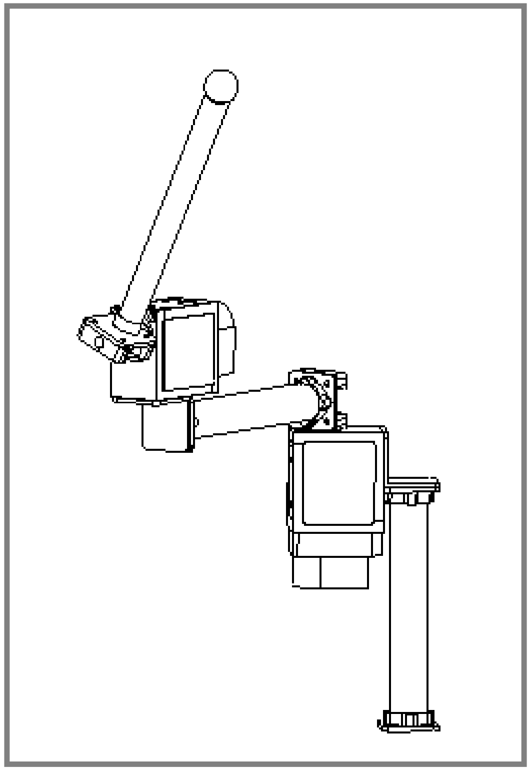


Simplified Model

Optimization Results



Design Study - Robot Link (1)



Horizontal Inverted Pendulum

Optimal Mechanical Design

Design Parameters x_i :

- **Mass**
- **Center of mass location**
- **Inertia about x direction**
- **Inertia about y direction**

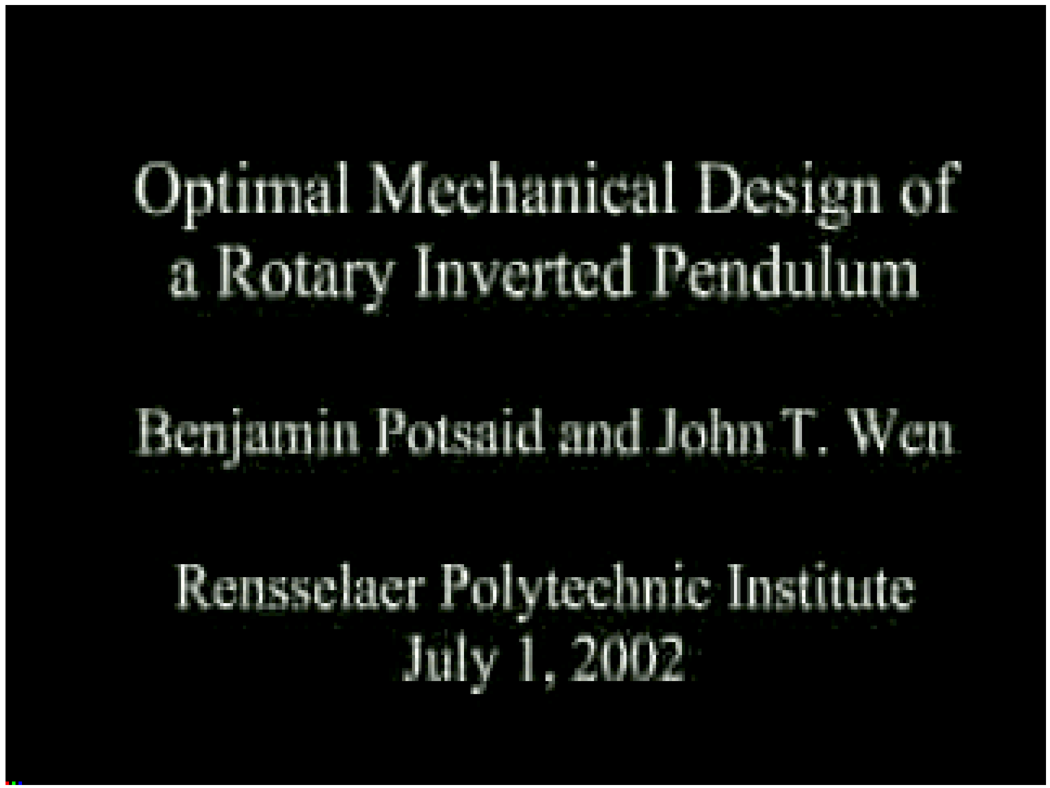
Optimize for

1. Controllability
2. Small unstable open loop pole

Design Study - Robot Link (1)

We can’t optimize both metrics simultaneously, so we get a family of design solutions.

Design Study - Robot Link (2)



Conclusion

- Mechanical Design and Control System Design cannot be done separately when high performance is required.
- Use the latest technologies and design tools to your advantage.
- Don’t forget about first principles and the basics.