Materials Selection and Design

For selection, one must establish a link between materials and function, with shape and process playing also a possibly important role (now ignored.)

**Areas of Design Concern**

- **Function**: support a load, contain a pressure, transmit heat, etc.
- **Shape**: determine the geometry of the component.
- **Process**: determine the manufacturing process.

**Materials Attributes**: physical, mechanical, thermal, electrical, economic, environmental.

**Objective**: make thing cheaply, light weight, increase safety, etc., or combinations of these.

**Constraints**: make thing cheaply, light weight, increase safety, etc., or combinations of these.

**What is negotiable but desired conditions?**

Following “Materials Selection in Mechanical Design”, M. Ashby

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**Examples of Materials Indices**

<table>
<thead>
<tr>
<th>Function, Objective, and Constraint</th>
<th>Index</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tie, minimum weight, stiffness</td>
<td>$E/\rho$</td>
</tr>
<tr>
<td>Beam, minimum weight, stiffness</td>
<td>$E^{1/2}/\rho$</td>
</tr>
<tr>
<td>Beam, minimum weight, strength</td>
<td>$\sigma^{2/3}/\rho$</td>
</tr>
<tr>
<td>Beam, minimum cost, stiffness</td>
<td>$E^{10/3}/C_\rho \cdot C_m$</td>
</tr>
<tr>
<td>Beam, minimum cost, strength</td>
<td>$\sigma^{2/3}/C_\rho$</td>
</tr>
<tr>
<td>Column, minimum cost, buckling load</td>
<td>$E^{15/3}/C_\rho$</td>
</tr>
<tr>
<td>Spring, minimum weight for given energy storage</td>
<td>$\sigma_n^{2/3}/E\rho$</td>
</tr>
<tr>
<td>Thermal insulation, minimum cost, heat flux</td>
<td>$1/(\alpha \cdot C_\rho \cdot \kappa)$</td>
</tr>
<tr>
<td>Electromagnet, maximum field, temperature rise</td>
<td>$\kappa \cdot C_\rho$</td>
</tr>
</tbody>
</table>

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**Design & Selection: Materials Indices**

Structural elements perform physical functions (carry load or heat, store energy...), and so they must satisfy certain functional requirements specified by the design, such as specified tensile load, max. heat flux, spring restoring force, etc.

**Material index** is a combination of materials properties that characterizes the performance of a material in a given application.

**Performance** of a structural element may be specified by the functional requirements, the geometry, and the material's properties.

**Performance**: $P = f(F) \cdot f(G) \cdot f(M)$

For **Optimum design**, we need to **maximize** or **minimize** the functional $P$.

Consider only the simplest cases where these factors form a separable equation:

$$P = f_1(F) \cdot f_2(G) \cdot f_3(M)$$

There is then enormous simplification and performance can be optimized by focusing on $f_3(M)$, which is the materials index

**Sw** safety factor should always be included!
Price and Availability of Materials

- Current Prices on the web:\(^a\): TRENDS
  - Short term: fluctuations due to supply/demand.
  - Long term: prices increase as deposits are depleted.

- Materials require energy to process them:
  - Energy to produce materials (GJ/ton)
    - Al: 237 (17)\(^b\)
    - PET: 103 (13)\(^c\)
    - Cu: 97 (20)\(^d\)
    - steel: 20\(^e\)
    - glass: 13\(^f\)
    - paper: 9\(^f\)

  - Cost of energy used in processing materials ($/GJ)\(^g\)
    - elect: 25
    - propane: 11
    - natural gas: 9

Recycling indicated in green.

Relative Cost (in $) of Materials

- Reference material: Rolled A36 carbon steel.
- Relative cost fluctuates less than actual cost over time.

Based on data in Appendix C, Callister, 6e.

$ = \frac{\text{$/kg}}{\text{ref material}}$

Materials Selection Examples in Mechanical Design with Separable Performance Factor

PERFORMANCE: functional needs, geometry, and materials index

\[ P = f_1(F) f_2(G) f_3(M) \]

\[ \text{Maximize Materials Index: } F \]

Example 1: Material Index for a Light, Strong, Tie-Rod

A Tie-rod is a common mechanical component.

Functional needs:
- Must carry tensile force, \(F\).
- NO failure: Stress must be less than \(\sigma_f\) (YS, UTS).
- \(L\) is usually fixed by design, can vary \(A\).
- While strong, need to be lightweight, or low mass.

Strength relation:
\[ F = \frac{\sigma_f}{\sigma} \leq \frac{F}{S} \]

Mass of rod:
\[ m = \rho L A \]

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\[ \text{Or Maximize Materials Index: } M = \frac{\sigma_f}{\rho} \]

\[ \text{Minimize for small } m \]

\[ m = \rho L A \]

\[ M = \frac{\sigma_f}{\rho} \]
Example 1: square rod (it’s all the same!)

- Carry F without failing; fixed initial length L.
- Strength relation: 
  \[ \frac{\sigma_f}{E} = \frac{F}{\frac{1}{2}b^2} \]
  \[ M = \rho b^2 \]
- Eliminate the "free" design parameter, c:
  \[ M = (FLS) \frac{\rho}{\sigma_f} \]
  specified by application

Maximize the Materials Performance Index:
(strong, light tension members)
\[ M_{\text{index}} = \frac{\sigma_f}{\rho} \]

Example 2: Material Index for a Light, Stiff Beam in Tension

- Bar must not lengthen by more than \( \delta \) under force F; must have initial length L.
- Stiffness relation: 
  \[ \frac{F}{E} \frac{\delta}{L} \]
  \[ m = \frac{pLc^2}{F} \]
- Eliminate the "free" design parameter, c:
  specified by application

Maximize the Materials Index:
(stiff, light tension members)
\[ M = \frac{E}{\rho} \]

Example 3: Material Index for a Light, Stiff Beam in Deflection

- Bar with initial length L must not deflect by more than \( \delta \) under force F.
- Stiffness relation: 
  \[ \frac{F}{E} \frac{\delta}{L} \]
  \[ m = \frac{b^2L^3}{12} \]
- Eliminate the "free" design parameter, A:
  specified by application

Maximize: 
\[ M = \frac{E^{1/2}}{\rho} \]

If only beam height can change (not A), then
\[ M = (E^{1/3}/\rho) \]
(Car door) 
\[ I \propto b^3w \]
If only beam width can change (not A), then
\[ M = (E/\rho) \]
**Performance of Square Beam vs. Fixed Height or Width**

- Light, Stiff Plate: $E/\rho$
- Light, Stiff Beam: $E^{1/2}/\rho$
- Light, Stiff Panel: $E^{1/3}/\rho$

**Example 4: Torsionally stressed shaft (Callister Chpt. 6)**

- Shaft must carry moment, $M_t$, with length $L$.
- Mass plus Twisting Moment, $M$: $\tau = \frac{2M_t}{\pi R^3}$
- Mass of bar: $m = \frac{\tau}{\sqrt{\rho}}$

**Eliminate the "free" design parameter, $R$:**

- Specified by application
- Minimize for small $M$

- Maximize the Material’s Index: (strong, light torsion members)

**Ashby Plot: Strength vs Density (on log scale)**

$M = 30$ has $1/3$ the mass of $M = 10$ (mass $\propto 1/M \propto \rho$).

$M = \frac{2\sqrt{3}}{3} \Rightarrow$

$\log \tau = \frac{3}{2} \log \rho + \frac{3}{2} \log M$

*All materials that lie on these lines will perform equally for strength-per-mass basis. However, each line has a different Materials $M$ index, or overall Performance $P$ index.*

**Data Overview: Strong & Light Tension/Torsion Members**

- Increasing $M$ for strong tension members
- Increasing $M$ for strong torsion members

*Adapted from Fig. 6.22, Callister 6e, Fig. 9.22 adapted from M.F. Ashby, Materials Selection in Mechanical Design, Butterworth-Heinemann Ltd., 1992.*
Strength vs Density

• Additional constraints may be added, such as strength having minimum value, e.g., \( \sigma_f > 300 \text{ GPa} \).

• Search area is then limited to the area in plot above all lines (if maximizing).

Other Material Indices: Cost factor

Considering mass

Maximizer: 
\[ M = \frac{\tau f}{\rho} \]

CRFP are best!

Considering mass

Maximizer: 
\[ M = \frac{\tau f}{\rho} \]

4340 Steel is best!

Other factors:

--require \( \sigma_f > 300 \text{ MPa} \).

--Rule out ceramics and glasses: \( K_{IC} \) too small.

• Maximize the Performance Index:

\[ P = \frac{\tau f^{2/3}}{\rho} \]

• Other factors:

Numerical Data: 

<table>
<thead>
<tr>
<th>Material</th>
<th>( \rho ) (Mg/m(^3))</th>
<th>( \tau_f ) (MPa)</th>
<th>( P ) (MPa)(^{2/3})m(^{-3})/Mg</th>
</tr>
</thead>
<tbody>
<tr>
<td>CFRE (v=0.65)</td>
<td>1.6</td>
<td>1140</td>
<td>73</td>
</tr>
<tr>
<td>GFRE (v=0.65)</td>
<td>2.0</td>
<td>1060</td>
<td>52</td>
</tr>
<tr>
<td>Al alloy (2024-T6)</td>
<td>2.8</td>
<td>300</td>
<td>16</td>
</tr>
<tr>
<td>Ti alloy (Ti-6Al-4V)</td>
<td>4.4</td>
<td>525</td>
<td>15</td>
</tr>
<tr>
<td>4340 steel (oil quench &amp; temper)</td>
<td>7.8</td>
<td>780</td>
<td>11</td>
</tr>
</tbody>
</table>

Data from Table 6.6, Callister 6e.

Details: Strong, Light Torsion Members

• Maximize the Performance Index: 

\[ P = \frac{\tau f^{2/3}}{\rho} \]

• Other factors: 

Numerical Data: 

<table>
<thead>
<tr>
<th>Material</th>
<th>( \rho ) (Mg/m(^3))</th>
<th>( \tau_f ) (MPa)</th>
<th>( M ) (MPa)(^{2/3})m(^{-3})/Mg</th>
</tr>
</thead>
<tbody>
<tr>
<td>CFRE (v=0.65)</td>
<td>73</td>
<td>80</td>
<td>112</td>
</tr>
<tr>
<td>GFRE (v=0.65)</td>
<td>52</td>
<td>40</td>
<td>76</td>
</tr>
<tr>
<td>Ti alloy (2024-T6)</td>
<td>16</td>
<td>15</td>
<td>93</td>
</tr>
<tr>
<td>4340 steel (oil quench &amp; temper)</td>
<td>11</td>
<td>5</td>
<td>46</td>
</tr>
</tbody>
</table>

Data from Table 6.7, Callister 6e.

Details: Strong, Low-Cost Torsion Members

• Minimize Cost: 

\[ \text{Cost Index} \sim m \$ \sim \frac{\$}{M} \] (since \( m \sim 1/M \))

Numerical Data: 

<table>
<thead>
<tr>
<th>Material</th>
<th>( M ) (MPa)(^{2/3})m(^{-3})/Mg</th>
<th>$ (\text{$/M}) \times 100</th>
</tr>
</thead>
<tbody>
<tr>
<td>CFRE (v=0.65)</td>
<td>73</td>
<td>112</td>
</tr>
<tr>
<td>GFRE (v=0.65)</td>
<td>52</td>
<td>76</td>
</tr>
<tr>
<td>Ti alloy (2024-T6)</td>
<td>15</td>
<td>748</td>
</tr>
<tr>
<td>4340 steel (oil quench &amp; temper)</td>
<td>11</td>
<td>46</td>
</tr>
</tbody>
</table>

Data from Table 6.7, Callister 6e.

• Lowest cost: 4340 steel (oil quench & temper)

• Need to consider machining, joining costs also.
Example 5: Material Index for a Cheap, Stiff Support Column
(From Ashby: Materials Selection in Mechanical Design)

A slender column of fixed initial length $L$ uses less material than a fat one; but must not be so slender that it buckles under load $F$.

- No buckling relation:  - Cost objective:

$$ F = \frac{Fl_0^2}{L^2} = \frac{\pi^2EI}{L^2} $$

Load less than Euler Load.

$$ C = \frac{mC_m - AL\rho C_m}{m} $$

$C$ is the cost/kg of (usually processed) material.

- Eliminate the "free" design parameter, $A$:

$$ C = \frac{4A}{\pi^2} \left( \frac{E}{L} \right)^{1/2} \left( \frac{\rho}{\pi^2} \right)^{1/2} $$

Maximize

Cheap, Stiff Beam

specified by application

minimize for small $m$

Example 6: Selecting a Slender but strong Table Leg
(Note this uses previous example from Ashby.)

Luigi Tavolina, furniture designer, conceives of a lightweight of table of simplicity, with a flat toughened glass top on slender, unbraced, cylindrical legs.

- Critical Elastic Load:  - Mass of leg:

$$ F = \frac{Fl_0^2}{L^2} = \frac{\pi^2EI}{L^2} $$

$$ m = \frac{\pi R^2 L}{2} $$

- Eliminate the "free" design parameter, $R$:

$$ m \geq \frac{4}\pi \left( \frac{\rho}{\pi^2} \right)^{1/2} \left( \frac{E}{L} \right)^{1/2} $$

Maximize

$$ M_1 = \frac{E}{\rho} $$

For slenderness, get $R$ for Critical Load Eq.:

$$ F = \frac{4\rho}{\pi} \left( \frac{\rho}{\pi^2} \right)^{1/2} \left( \frac{E}{L} \right)^{1/2} $$

$$ M_2 \geq E $$

2 indices to meet

Example 5 (cont)

Material indices: $M_1 \geq \frac{E^{1/2}}{\rho}$ and $M_1 = E$

With cost considered, now polymers and metals are useful!

With wood:

- Wood is good choice.
- So is composite CFRP (higher $E$).
- Ceramic meets stated design goals, but are brittle

Cheap, Stiff Support

Cheap, Stiff Beam
Example 7: Elastic Recovery of Springs

Recall from Hooke’s Law and Resilience, \( U = \frac{1}{2} \sigma^2 \). We wish to maximize this, but the spring will be damaged if \( \sigma > \sigma_y \), \( U = \frac{1}{2} \sigma_y^2 / E \).

(Torsion bars and lead spring are less efficient than axial springs because some of the material is not fully loaded, for instance, the neutral axis it is not loaded at all!)

\[ U = \frac{1}{2} \sigma_y^2 / E \]

Can show that \( U = (\sigma_y^2 / E) / 18 \) Addition constraint can be added.

- If in-service, a spring under goes deflection of \( d \) under force \( F \), then \( \sigma_y^2 / E \) has to be high enough to avoid permanent set (a high resilience!).

- For this reason spring materials are heavily SS-strengthening and work-hardening (e.g., cold-rolled single-phase brass or bronze), SS plus precipitation strengthening (spring steel).

- Annealing any spring material removes work-hardening, or cause precipitation to coarsen, reducing YS and making materials useless as a spring!

Example 8: Safe Pressure Vessel

Uses info from leak-before-fail example.

- Choose \( t \) so that at working pressure, \( p \), the stress is less than \( \sigma_y \).

- Check (by x-ray, ultrasonics, etc.) that no cracks greater than \( 2a \) are present; then the stress required to active crack propagation is

\[ \sigma = \frac{K_I}{\sqrt{t}} \]

- Tolerable crack size is maximized by choosing largest

\[ M_1 = K_I / \sigma_y \]

- Large pressure vessels cannot always be tested for cracks and stress testing is impractical. Cracks grow over time by corrosion or cyclic loading (cannot be determined by one measurement at start of service).

- Leak-before-break criterion (leaks can be detected over lifetime)

\[ \frac{K_I}{\sqrt{t}} \leq \frac{1}{\sqrt{t}} \]

- Wall thickness was designed to contain pressure w/o yielding, so \( \sigma = \sigma_y \).

- Two equations solved for maximum pressure gives

\[ M_2 = (K_I \sigma_y / \sigma) \]

- Largest \( M_2 \) and \( M_3 \) for smallest \( \sigma \). FOOLISH for pressure vessel.

- Wall thickness must be thin for lightness and economy.

- Thinnest wall has largest yield stress, so \( M_3 = \sigma_y \).

Yield-before-break

\[ M_1 = K_I / \sigma_y \]

Leak-before-break

\[ M_2 = (K_I \sigma_y / \sigma) \]

Thin wall, strong

\[ M_3 = \sigma_y \]

- Large pressure vessels are always made of steel.

- Models are made of Cu, for resistance to corrosion.

- Check that \( M_3 \) favors steel.

- \( M > 100 \) MPa eliminates Al.
Optimal Magnet Coil Material (see CDROM)

- High magnetic fields permit study of:
  - electron energy levels,
  - conditions for superconductivity,
  - conversion of insulators into conductors.

- Estimation of Lorentz stress can exceed the material strength.
  - Lorentz stress can melt the coil.
  - Intense resistive heating can melt the coil.

Technical Challenges:
- Applied magnetic field: $H = N I / L$
- Lorentz "hoop" stress: $\sigma = \frac{H_0 R}{A} \left( \leq \frac{H_0}{S} \right)$
- Resistive heating: $\Delta T = \frac{2 \rho_c \Delta T_{max}}{A^2 c_v}$

Magnet Coil: Performance Index

- Mass of coil: $m = \rho_d AL$
- Applied magnetic field: $H = N I / L$

- Eliminate "free" design parameters $A$, $I$ from the stress & heating equations (previous slide):
  - Stress requirement: $\frac{H^2}{m} \leq \frac{1}{2 \pi R^2 L} \frac{\mu_o N}{\rho_d}$
  - Heating requirement: $\frac{H^2}{m} \leq \frac{H_{max}^2}{2 \pi R L} \frac{1}{\rho_e}$

Magnet Coil: Cost Index

- Relative cost of coil: $S = \frac{M}{\rho_d}$
- Applied magnetic field: $H = N I / L$

- Eliminate $M$ from the stress & heating equations:
  - Stress requirement: $\frac{H^2}{M} \leq \frac{1}{2 \pi R L} \frac{\mu_o N}{\rho_e}$
  - Heating requirement: $\frac{H_{max}}{M} \leq \frac{1}{2 \pi R L} \frac{1}{\rho_o}$

Fractured magnet coil (Photo by J. Bevk, Los Alamos National Labs, 1995).
Indices For A Coil Material

<table>
<thead>
<tr>
<th>Material</th>
<th>ηv</th>
<th>ηv</th>
<th>$ \sigma_c$</th>
<th>$\phi_c$</th>
<th>$\psi_c$</th>
<th>$\Delta_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1020 steel (an)</td>
<td>395</td>
<td>7.85</td>
<td>0.8</td>
<td>486</td>
<td>1.60</td>
<td>50</td>
</tr>
<tr>
<td>1100 Al (an)</td>
<td>90</td>
<td>2.71</td>
<td>12.3</td>
<td>904</td>
<td>0.29</td>
<td>33</td>
</tr>
<tr>
<td>7075 Al (T6)</td>
<td>572</td>
<td>2.80</td>
<td>13.4</td>
<td>960</td>
<td>0.52</td>
<td>20</td>
</tr>
<tr>
<td>11000 Cu (an)</td>
<td>220</td>
<td>8.89</td>
<td>7.9</td>
<td>385</td>
<td>0.17</td>
<td>25</td>
</tr>
<tr>
<td>17200 Bi-Cu (sl)</td>
<td>475</td>
<td>8.25</td>
<td>51.4</td>
<td>420</td>
<td>0.57</td>
<td>58</td>
</tr>
<tr>
<td>71500 Cu-Al (fr)</td>
<td>380</td>
<td>9.64</td>
<td>52.9</td>
<td>380</td>
<td>3.75</td>
<td>43</td>
</tr>
<tr>
<td>Pt</td>
<td>145</td>
<td>21.5</td>
<td>1.84</td>
<td>132</td>
<td>1.06</td>
<td>7</td>
</tr>
<tr>
<td>Ag (an)</td>
<td>170</td>
<td>19.5</td>
<td>271</td>
<td>235</td>
<td>0.15</td>
<td>16</td>
</tr>
<tr>
<td>Ni 200</td>
<td>462</td>
<td>8.89</td>
<td>31.4</td>
<td>458</td>
<td>0.95</td>
<td>52</td>
</tr>
</tbody>
</table>

Avg. values used. an = annealed; T6 = heat treated & aged; sl = solution heat treated; fr = hot rolled

Indices For A Coil Material

- Lightest for a given H: 7075 Al (T6) $\leftrightarrow$ P1
- Lightest for a given H($\Delta_t$): 1100 Al (an) $\leftrightarrow$ P2
- Lowest cost for a given H: 1020 steel (an) $\leftrightarrow$ C1
- Lowest cost for a given H($\Delta_t$): 1020 steel (an) $\leftrightarrow$ C2

Material costs fluctuate but rise over long term as:
- rich deposits are depleted,
- energy costs increase.

Recycled materials reduce energy use significantly.

Materials are selected based on:
- performance or cost indices.

Examples:
- design of minimum mass, maximum strength of:
  - shafts under torsion,
  - bars under tension,
  - plates under bending,
- selection to optimize more than one property:
  - leg slenderness and mass,
  - pressure vessel safety,
  - material for a magnet coil (see CD-ROM).