Today (9/11/01)

- Announcements:
  - The inertia term *I* in exercise 2 should be $I_m N^2 + I + m l_g^2$ (make sure you use the right value for Project 1)
  - The nonlinear step response would tend to infinity if input torque is larger than the maximum gravity torque $m l_g g$ and to a steady state if it's less.
- You should be making progress on Project 1 (I'd like to see by this Friday major progress on parts 1-4, and some progress on parts 7 and 9)
- I'll be out of town this Thursday, office hour rescheduled to Wednesday 2-3.

Lecture today

- **Response of continuous-time LTI systems (3.3-3.5)**

Dynamic Response

- Response vs. pole location
- Second order system with no zero
- Time domain performance specification
- Effect of additional zeros and poles

- Ref: 3.3-3.5

Response vs. Pole Locations

$$\text{Consider a first order system}: H(s) = \frac{1}{s+s}$$

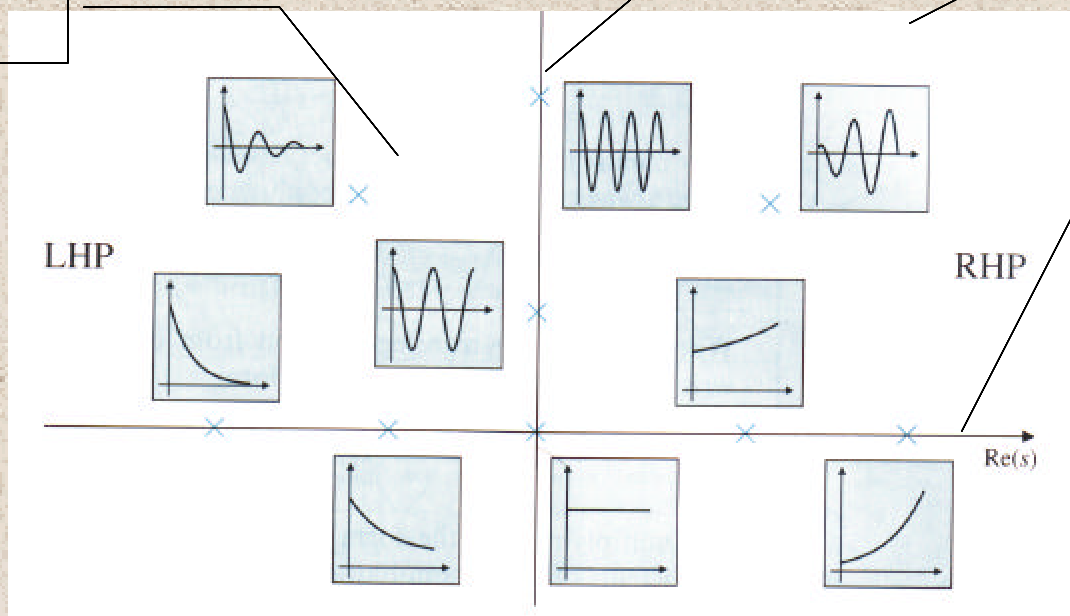
$$\text{Impulse response}: h(t) = e^{-st}1(t)$$

$$\text{Step response}: y_s(t) = \frac{1-e^{-st}}{s}1(t)$$

Marginally stable

unstable

stable



Systems with real poles only

Second order system

- Classical control system design draws based on second order system with no zero

$$H(s)=\frac{w_n^2}{s^2+2zw_n s+w_n^2}$$

w_n : undamped natural frequency (rad/s)

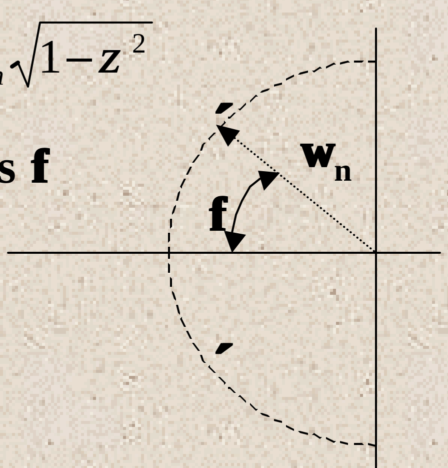
z : damping ratio (in %)

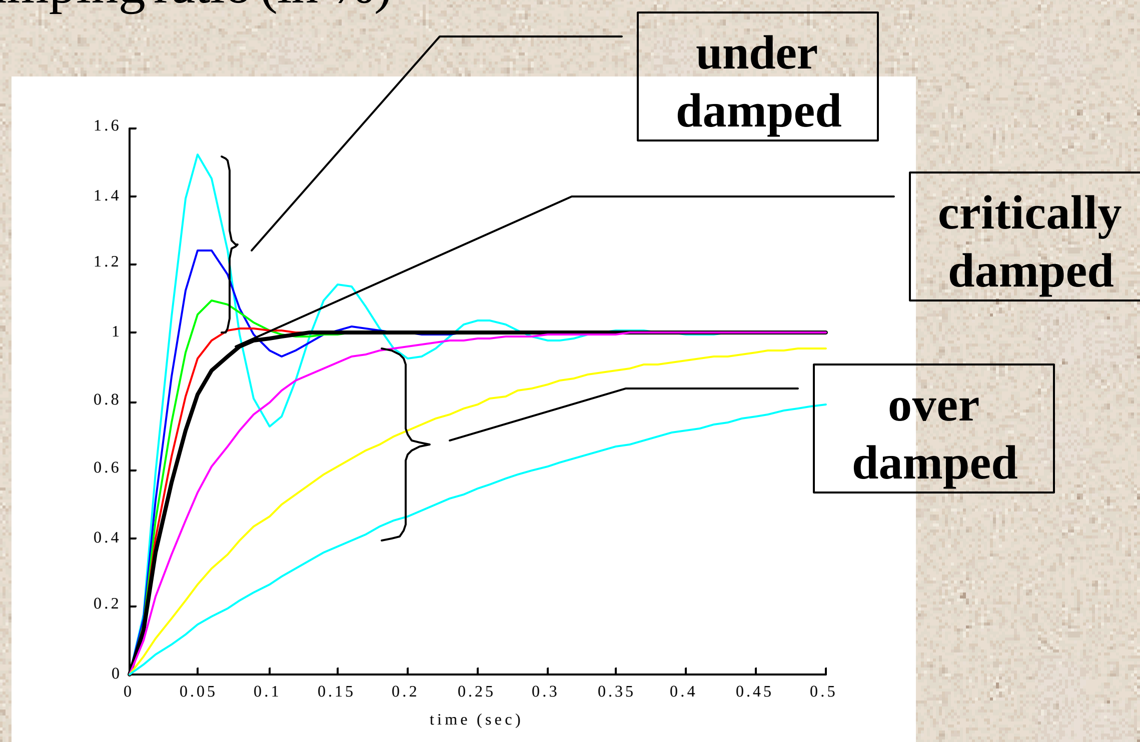
$$h(t)=\frac{w_n^2}{w_d}e^{-stt}\sin(w_d t)1(t)$$

$$y(t)=1-e^{-stt}(\cos w_d t+\frac{s}{w_d}\sin w_d t)$$

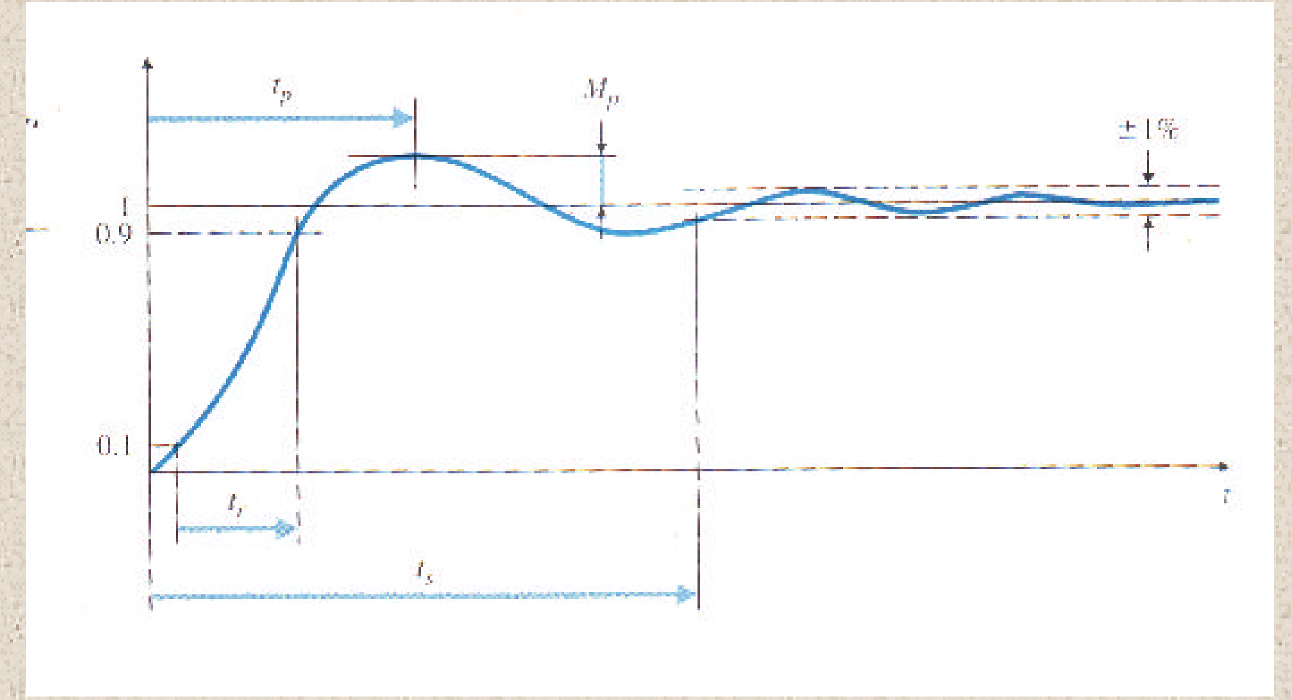
$$w_d=w_n\sqrt{1-z^2}$$

z=cos f





Time Domain Performance



tstats.m

t_r : rise time (time from 10% to 90% of final value)

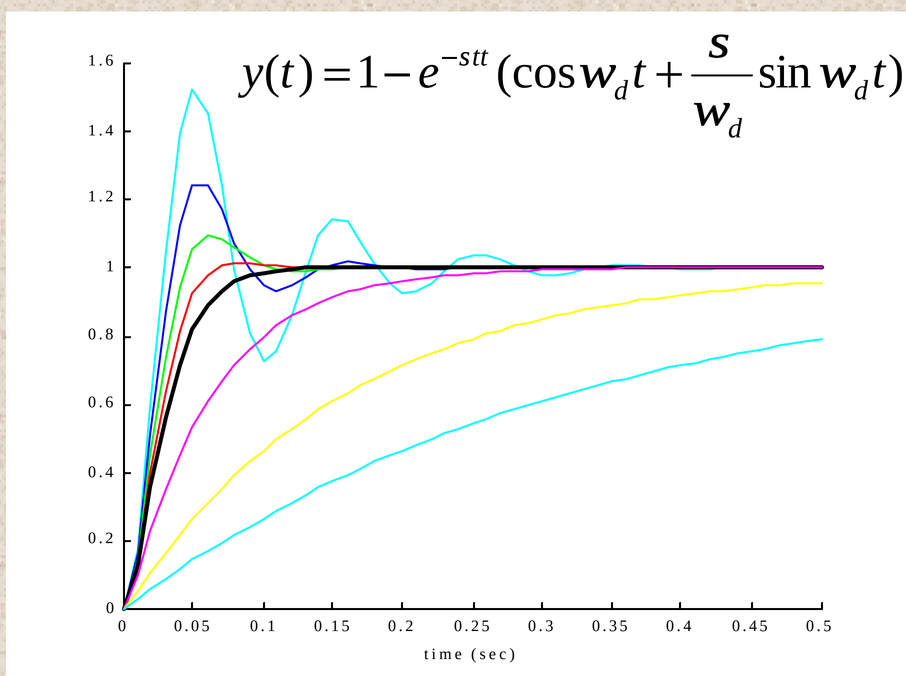
t_s : settling time (time to converge within 99% of final value)

M_p : Overshoot (max overshoot/final value, as %)

t_p : peak time (time to reach max overshoot)

Time Domain Spec vs. Poles

Use 2nd order system response to generate rule of thumb



- **Rise time:choose ζ=.5 as an average** $t_r \cong \frac{1.8}{\omega_n}$
- **Peak time: set** $\dot{y}(t) = 0$ **and solve for t_p and M_p**

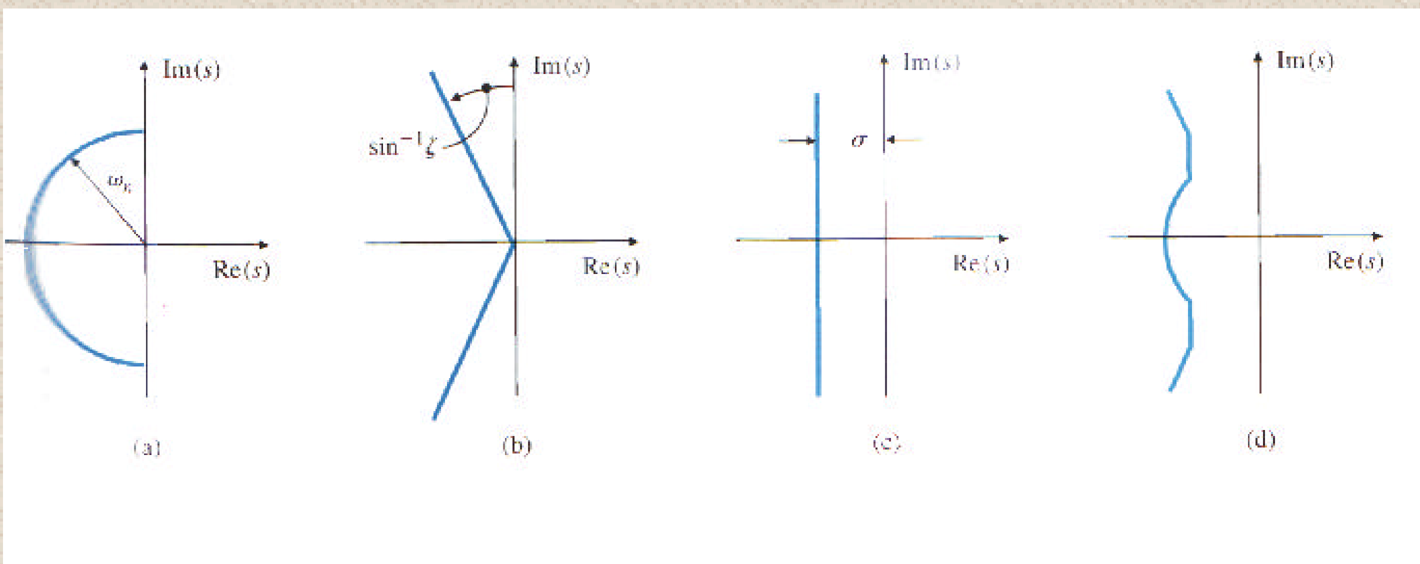
$$t_p \cong \frac{\pi}{\omega_d} \quad M_p \cong e^{-\pi\zeta/\sqrt{1-\zeta^2}}$$

- **1% settling time:** $t_s \cong \frac{4.6}{\sigma}$

Rule of Thumb for Pole Locations

Given time domain specifications: t_r , M_p , t_s , choose target pole locations as:

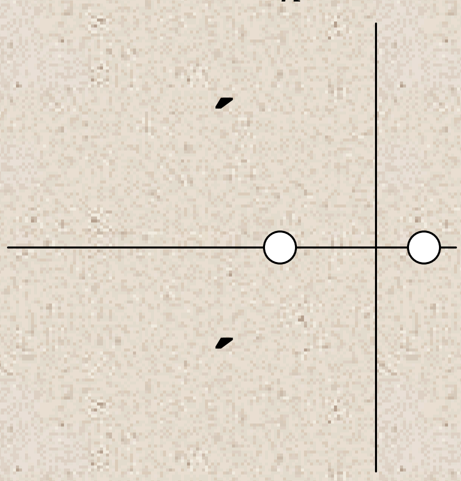
$$w_n \geq \frac{1.8}{t_r} \qquad z \geq z(M_p) \qquad s \geq \frac{4.6}{t_s}$$

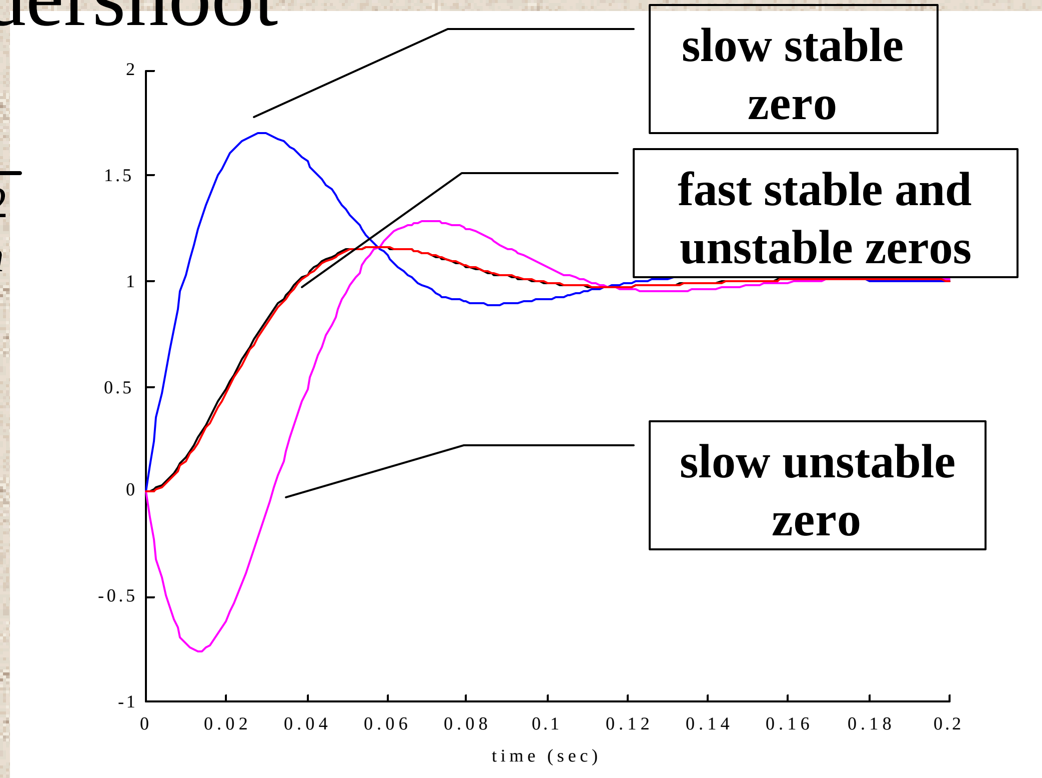


Effect of Additional Zeros

- Zero within system bandwidth strongly affects response
- Stable zero increases overshoot, unstable zero gives rise to undershoot

$$H(s) = \frac{w_n^2 (s/a+1)}{s^2 + 2zw_n s + w_n^2}$$



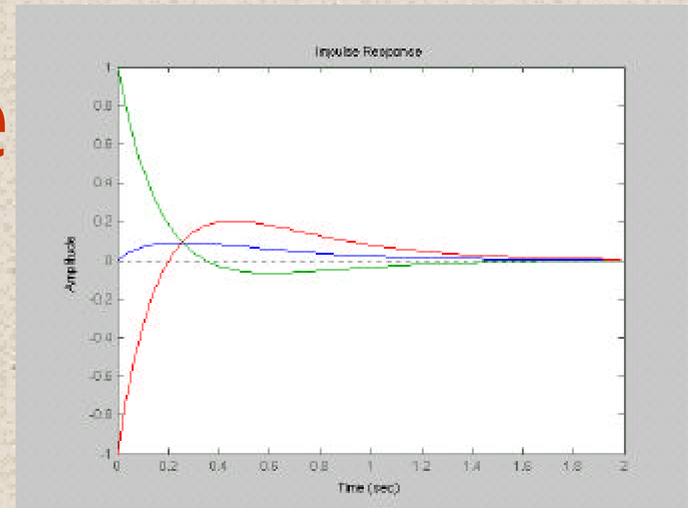


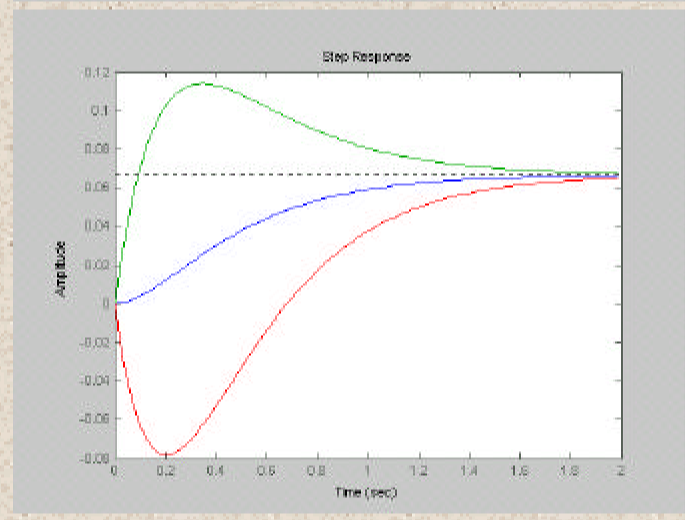
Example

$$\frac{1}{(s+3)(s+5)} = \frac{1}{2(s+3)} - \frac{1}{2(s+5)}$$

$$\frac{s+1}{(s+3)(s+5)} = -\frac{1}{(s+3)} + \frac{2}{(s+5)}$$

$$\frac{-s+1}{(s+3)(s+5)} = \frac{2}{(s+3)} - \frac{3}{(s+5)}$$



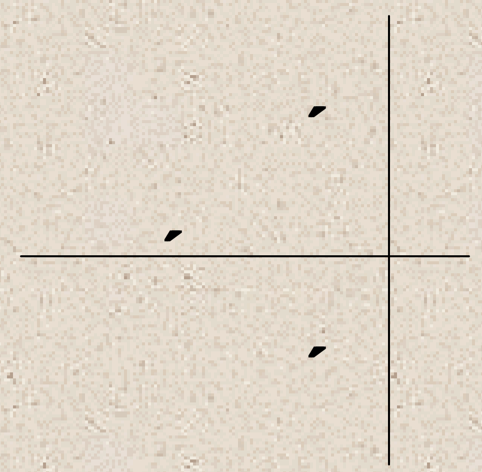


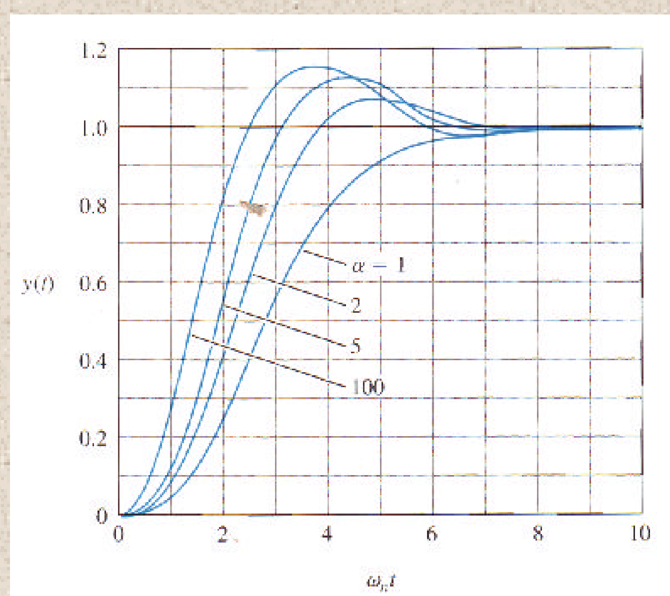
Same modes, but relative contribution is changed by the zero

Effect of Additional Pole

- An additional pole (within factor of 4) of fastest of the other two poles will increase rise time and overshoot.

$$H(s) = \frac{1}{\left(s/\alpha\zeta\omega_n + 1\right)\left[(s/\omega_n)^2 + 2\zeta\,(s/\omega_n) + 1\right]}$$





Summary

- General relationship between zero/pole locations and time response is complicated.
- Performance specification typically in time domain but control design typically specifies pole locations.
- Rule of thumb based on second order systems with no zero.
- Watch out for additional zeros and poles.

Exercise 3

- Determine if the linearized system is stable (as a function of θ_d). When the system is stable, find the steady state value.
- For $\theta_d=0$, find the rise time, peak time, settling time, and overshoot. Compare the values with the formula for second order systems.